Diffractive and total pp cross sections at the LHC and beyond

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Abstract. Results on factorization breaking in soft and hard hadron-hadron collisions, photo-production and deep inelastic scattering exhibit a universal behavior in a renormalization model where diffraction is mediated by a saturated colorless exchange with vacuum quantum numbers. Using this model, diffractive and total cross sections are predicted for LHC energies.

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1. INTRODUCTION

Diffractive and total $p(\bar{p})$-p cross sections at high energies are dominated by Pomeron ($I\!P$) exchange, and for a Pomeron intercept $\alpha(0) = 1+\varepsilon$ the s-dependence is given by [1]-[3]:

$$(d\sigma_{el}/dt)_{t=0} \sim (s/s_o)^{2\varepsilon}, \;\; \sigma_t = \beta^2_{I\!Ppp}(0)\cdot(s/s_o)^{\varepsilon}, \;\; \text{and } \sigma_{sd} \sim (s/s_o)^{2\varepsilon}, \tag{1}$$

where s is the collision energy squared and s_o is a scale parameter. Such behavior would violate unitarity at high energies, when the elastic and / or single diffractive (SD) cross section would exceed the total cross section. In the case of SD, CDF results at $\sqrt{s} =$ 540 GeV [1800 GeV] showed that violation of unitarity is avoided by a suppression of $\sigma_{sd}(s)$ by a factor of $\mathscr{O}(5)$ [factor of $\mathscr{O}(10)$] relative to Regge expectations (see Ref. [2]).

In the present paper[1], we describe the total cross section in a model based on a saturated Froissart bound above $s = s_F$, predicting a $\ln^2 s$ dependence for σ_t at $s \geq s_F$,

$$\sigma_t(s > s_f) = \sigma_t(s_F) + (\pi/s_o)\cdot\ln^2(s/s_F), \tag{2}$$

where the parameters s_F and s_o are experimentally determined from available SD data.

2. DIFFRACTION

In Regge theory, the differential SD cross section is given by:

$$\frac{d^2\sigma_{sd}(s,\xi,t)}{d\xi\, dt} = (\beta^2_{I\!Ppp}/16\pi)\xi^{1-2\alpha(t)}\cdot\beta_{I\!Ppp}(0)g(t)\left(s'/s_\circ\right)^{\varepsilon}, \tag{3}$$

[1] This paper contains excerpts from the paper by this author presented in Ref. [3], and is updated to include a prediction of σ_t at $\sqrt{s} = 8$ TeV, in addition to $\sqrt{s} = 7$ and 14 TeV.

where $g(t)$ is the triple-$I\!P$ coupling, $f_{I\!P/p}(\xi,t)$ the *Pomeron flux* and $\sigma^{I\!Pp}(s',t)$ the $I\!Pp$ total cross section at the sub-energy squared s'. In Ref. [2], the unitarity violation arising in the theory from the $s^{2\varepsilon}$ dependence of the SD cross section was resolved in the renormalization model by interpreting the Pomeron flux as the production probability of the rapidity gap and *renormalizing* the integrated probability over all phase space in ξ and t to unity above the energy at which it saturates to unity. In effect, the renormalization procedure eliminates double-counting from overlapping rapidity gaps and is technically implemented by dividing the expression of the differential SD cross section by the Pomeron flux integral at energies above the saturation energy.

In addition to averting unitarity violation by slowing down the growth of the diffractive cross section with increasing energy, the renormalization model resolved an outstanding energy scale issue. From Eq. (1), it is seen that $\beta^2_{I\!Ppp}(0) \sim s_o^{\varepsilon}$, and therefore in the diffractive cross section of Eq. (3) the Pomeron flux contains a scale factor s_o^{ε}, and the $I\!P$-p cross section contains a factor $s_o^{\varepsilon} \cdot [s_o^{\varepsilon/2} \cdot g(t) \cdot s_o^{-\varepsilon}] = s_o^{\varepsilon/2} \cdot g(t)$. Consequently, only the product $g(t) \cdot s_o^{\varepsilon/2}$ can be determined from the measurement of σ_{sd}.

The $g(t)$-s_o entanglement was resolved in Ref. [2] by using for s_o a value determined from results on $\sigma_{sd}(s)$. It was argued that the observed *knee* in the cross section at $\sqrt{s}_{\rm knee} = 22$ GeV (see Ref. [2]) occurs precisely at the energy at which the Pomeron flux integral saturates to unity. Since this integral depends on both s and s_o, the measurement of $\sqrt{s}_{\rm knee}$ was used to determine s_o. The value of s_o was found to be $s_o = 1 \pm 0.4$ GeV^2.

By a change of variables from ξ to M^2, using $\xi \approx M^2/s$, Eq. (3) becomes:

$$\frac{d^2\sigma_{sd}(s,M^2,t)}{dM^2dt} = \left[\frac{\sigma_\circ}{16\pi}\sigma_\circ^{I\!Pp}\right] \frac{s^{2\varepsilon}}{N(s,s_o)} \frac{e^{bt}}{(M^2)^{1+\varepsilon}}, \tag{4}$$

where $b = b_0 + 2\alpha' \ln\frac{s}{M^2}$ is the slope parameter of the t-distribution and $N(s,s_o)$ the integrated Pomeron flux,

$$N(s,s_o) \equiv \int_{\xi_{\rm min}}^{\xi_{\rm max}} d\xi \int_{t=0}^{-\infty} dt\, f_{I\!P/p}(\xi,t) \overset{s\to\infty}{\to} \sim s_o^{\varepsilon} \frac{s^{2\varepsilon}}{\ln s}. \tag{5}$$

The asymptotic form of the differential cross section, obtained from Eqs. (4) and (5),

$$\frac{d^2\sigma_{sd}(s,M^2,t)}{dM^2dt} \overset{s\to\infty}{\to} \sim \ln s \frac{e^{bt}}{(M^2)^{1+\varepsilon}}. \tag{6}$$

illustrates that division by the flux integral replaces $s^{2\varepsilon}$ by $\ln s$, preserving unitarity.

The extraction of s_o from the data in Ref. [2] was performed using $\varepsilon = 0.115 \pm 0.008$, which is the average of the CDF measurements at $\sqrt{s} = 540$ GeV and 1800 GeV. There is, however, a strong correlation between ε and s_o through the relationship displayed in Eq. 5. Using the more accurate value of ε obtained by R. J. M. Covolan, J. Montanha and K. Goulianos [4] from an *eikonal* global fit to $p^{\pm}p$, $K^{\pm}p$ and $\pi^{\pm}p$ cross sections, $\varepsilon = 0.104 \pm 0.002$, yields:

$$s_o^{\rm CMG} = (3.7 \pm 1.5) \text{ GeV}^2. \tag{7}$$

This value is used below in a numerical evaluation of $\sigma_t(s)$ based on Eq. 2.

3. TOTAL CROSS SECTION

Theoretical models predicting the total cross section at the LHC must satisfy all unitarity constraints. Various unitarization procedures are employed, and available accelerator and cosmic ray data on soft processes have been used to tune parameters in the models before fitting total cross section data and extrapolating to LHC energies. However, while a rise of the total cross section from Tevatron to LHC is generally obtained, the predictions for LHC are spread over a wide range. For example, in Ref. [3], several authors predict total cross sections at $\sqrt{s} = 14$ TeV ranging from 90 to 250 mb. The large disparity among these cross section predictions is mainly due to theoretical uncertainties emanating from the unitarization method employed and/or from the values and uncertainties of the parameters used in the models. The predictive power of our inherently unitarized approach based on the *analytic* expression of Eq. 2 is only limited by the uncertainties propagated from the statistical and systematic uncertainties of the two experimentally determined parameters s_F and s_o.

In a recent paper [6], a parton model approach to diffraction was introduced as a special phenomenological interpretation of the parton model for the Pomeron in QCD described by E. Levin in Ref. [5]. The parton model yields formulas similar to those of Regge theory. In Ref. [6], interpreting the term which is equivalent to the Pomeron flux as a gap formation probability leads naturally to a QCD concept of the renormalization procedure as eliminating double-counting from multiple exchanges of wee-partons (lowest energy partons in a partonic cascade), which provide the color-shield to a primary partonic exchange enabling the formation of diffractive rapidity gap.

Returning to Eq. (2), the saturation of the Froissart bound occurs in the *wee-parton* exchange governed by the value of the scale parameter s_o that enters in the diffractive cross section in Eq. (3). Interpreting s_o as the mass-squared of a partonic glueball-like *object* that is exchanged, and inserting it into the Froissart formula in place of the traditionally used m_π^2, should saturate the bound at the collision energy of:

$$\sqrt{s_F} = 22 \text{ GeV}, \tag{8}$$

which is the value obtained in Ref. [2] from the *knee* in the $\sigma_{sd}(s)$ distribution.

Predicting the total cross section at the LHC using Eq. (2) requires knowledge of $\sigma(s_F)$. However, the cross section at $\sqrt{s_F} = 22$ GeV has substantial Reggeon exchange contributions, and also contributions from the interference between the nuclear and Coulomb amplitudes. A complete description would have to take all these contributions into consideration, using Regge theory amplitudes to describe Reggeon exchanges, and dispersion relations to obtain the real part of the amplitude from the measured total cross sections up to Tevatron energies. Here, we follow a strategy that bypasses all these hurdles. For completeness, we outline below all steps in our cross section evaluation process:

(i) use the Froissart formula as a *saturated* rather than an upper bound;

(ii) Eq. (2) should then describe the cross section above the *knee* in σ_{sd} vs. $\sqrt{s}$, which occurs at $\sqrt{s_F} = 22$ GeV (Fig. 1 in Ref. [2]), and therefore should be valid at the Tevatron at $\sqrt{s} = 1800$ GeV;

(iii) set $m^2 = s_o/(\hbar c)^2 \approx s_o/0.389$ GeV^2·mb, where $s_o = s_o^{\rm CMG} = (3.7 \pm 1.5)$ GeV^2 – see Eq. (7);

(iv) note that contributions from Reggeon exchanges at $\sqrt{s} = 1800$ GeV are negligible, as can be verified from the global fit of Ref. [4];

(v) obtain the total cross section at the LHC as:

$$\sigma_t^{\rm LHC} = \sigma_t^{\rm CDF} + \frac{\pi}{s_o} \cdot \left(\ln^2 \frac{s^{\rm LHC}}{s_F} - \ln^2 \frac{s^{\rm CDF}}{s_F} \right). \tag{9}$$

Using the CDF total cross section of 80.03 ± 2.24 mb at $\sqrt{s} = 1.8$ TeV yields:

TABLE 1. Predicted σ_t^{pp}

$\sqrt{s}$	σ_t^{pp} (mb)
7 TeV	98 ± 8
8 TeV	100 ± 8
14 TeV	109 ± 12

The result for $\sqrt{s} = 14$ TeV falls within the range of cross sections predicted by the various authors in Ref. [3], and is in good agreement with $\sigma_t^{\rm CMG} = 114 \pm 5$ mb obtained by the global fit of Ref. [4].

4. SUMMARY

The total pp cross section at the LHC is predicted in a phenomenological approach that obeys all unitarity constraints. The approach is based on a saturated Froissart bound above a pp collision energy squared $s = s_F$, leading to an analytic $\ln^2 s$ dependence for the total cross section, $\sigma_t = (\pi/s_o) \cdot \ln^2(s/s_F)$. The formula contains two scale parameters, s_F and s_o, which are experimentally determined from presently available SD cross section data. A total cross section of $\sigma_t = 98 \pm 8$ mb , 100 ± 10 mb and 109 ± 12 mb is predicted for a pp collision energy of $\sqrt{s} = 7$, 8, and 14 TeV, respectively.

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