

# Predictions of Diffractive and Total Cross Sections at LHC Confirmed by Measurements



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  - CMS PAS <http://cds.cern.ch/record/1547898/files/FSQ-12-005-pas.pdf>
  - DIS13 [http://pos.sissa.it/archive/conferences/191/067/DIS%202013\\_067.pdf](http://pos.sissa.it/archive/conferences/191/067/DIS%202013_067.pdf)
  - MPI@LHC 2013 summary: <http://arxiv.org/abs/1306.5413>
  - CTEQ Workshop, "QCD tool for LHC Physics: From 8 to 14 TeV, what is needed and why" *FINAL, 14 November, 2013*

# Basic and combined diffractive processes

acronym basic diffractive processes

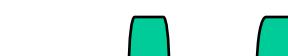
**SD<sub>̄p</sub>**  $\bar{p}p \rightarrow \bar{p} + \text{gap} + [p \rightarrow X_p],$



**SD<sub>p</sub>**  $\bar{p}p \rightarrow [\bar{p} \rightarrow X_{\bar{p}}] + \text{gap} + p,$



**DD**  $\bar{p}p \rightarrow [\bar{p} \rightarrow X_{\bar{p}}] + \text{gap} + [p \rightarrow X_p],$



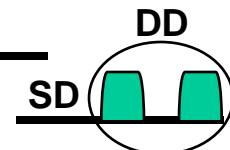
**DPE**  $\bar{p}p \rightarrow \bar{p} + \text{gap} + X_c + \text{gap} + p,$

2-gap combinations of SD and DD

**SDD<sub>̄p</sub>**  $\bar{p}p \rightarrow \bar{p} + \text{gap} + X_c + \text{gap} + [p \rightarrow X_p],$



**SDD<sub>p</sub>**  $\bar{p}p \rightarrow [\bar{p} \rightarrow X_{\bar{p}}]\text{gap} + X_c + \text{gap} + p.$



4-gap diffractive process-Snowmass 2001- <http://arxiv.org/pdf/hep-ph/0110240>

$\Delta\eta_1$

$\Delta\eta'_1$

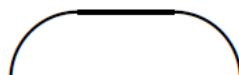
$\Delta\eta_2$

$\Delta\eta'_2$

$\Delta\eta_3$

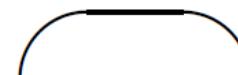
$\Delta\eta'_3$

$\Delta\eta_4$



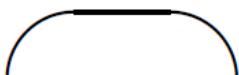
$\eta'_1$

$t_1$



$\eta_2$

$t_2$



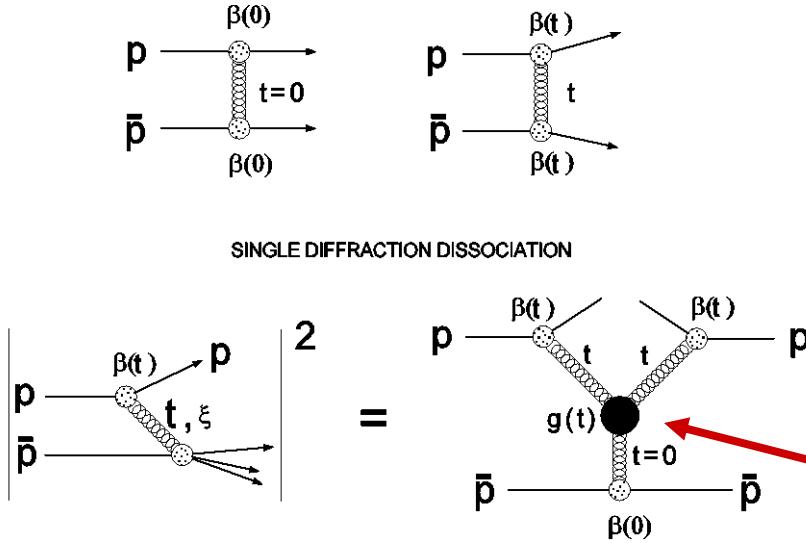
$\eta'_3$

$t_3$

$t_4$

# Regge theory – values of $s_0$ & $g_{PPP}$ ?

KG-PLB 358, 379 (1995)



- Parameters:
- $s_0$ ,  $s_0'$  and  $g(t)$
  - set  $s_0' = s_0$  (universal IP)
  - determine  $s_0$  and  $g_{PPP}$  – how?

$$\alpha(t) = \alpha(0) + \alpha' t \quad \alpha(0) = 1 + \epsilon$$

$$\sigma_T = \beta_1(0) \beta_2(0) \left( \frac{s}{s_0} \right)^{\alpha(0)-1} = \sigma_0^{p\bar{p}} \left( \frac{s}{s_0} \right)^{\epsilon} \quad (1)$$

$$\begin{aligned} \frac{d\sigma_{el}}{dt} &= \frac{\beta_1^2(t) \beta_2^2(t)}{16\pi} \left( \frac{s}{s_0} \right)^{2[\alpha(t)-1]} \\ &= \frac{\sigma_T^2}{16\pi} \left( \frac{s}{s_0} \right)^{2\alpha't} F^4(t) \approx \frac{\sigma_T^2}{16\pi} e^{b_{el}(s)t} \end{aligned} \quad (2)$$

$$F^4(t) \approx e^{b_{0,el}t} \Rightarrow b_{el}(s) = b_{0,el} + 2\alpha' \ln \left( \frac{s}{s_0} \right) \quad (3)$$

$$\begin{aligned} \frac{d^2\sigma_{sd}}{dt d\xi} &= \frac{\beta_1^2(t)}{16\pi} \xi^{1-2\alpha(t)} \left[ \beta_2(0) g(t) \left( \frac{s'}{s'_0} \right)^{\alpha(0)-1} \right] \\ &= f_{p/p}(\xi, t) \sigma_T^{p\bar{p}}(s', t) \end{aligned} \quad (4)$$

# A complication ... → Unitarity!

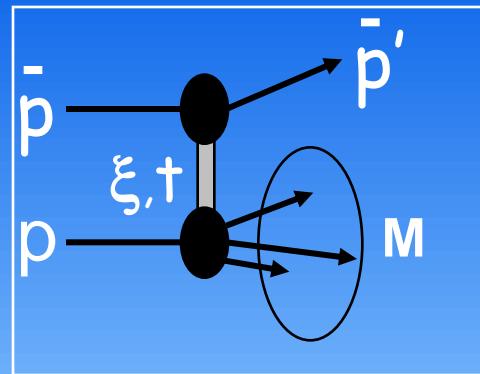
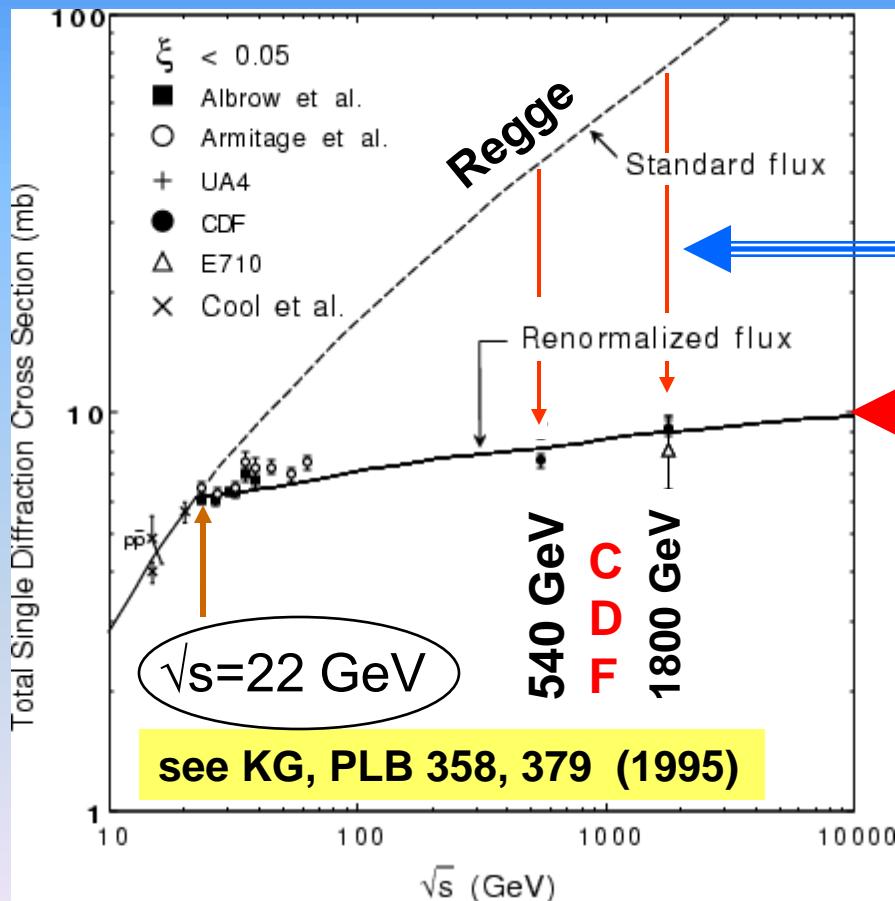
$$\left(\frac{d\sigma_{el}}{dt}\right)_{t=0} \sim \left(\frac{s}{s_o}\right)^{2\epsilon}, \quad \sigma_t \sim \left(\frac{s}{s_o}\right)^\epsilon, \quad \text{and} \quad \sigma_{sd} \sim \left(\frac{s}{s_o}\right)^{2\epsilon}$$

- $\sigma_{sd}$  grows faster than  $\sigma_t$  as  $s$  increases \*  
→ unitarity violation at high  $s$   
(similarly for partial x-sections in impact parameter space)
- the unitarity limit is already reached at  $\sqrt{s} \sim 2 \text{ TeV}$  !
- need unitarization

\* similarly for  $(d\sigma_{el}/dt)_{t=0}$  w.r.t.  $\sigma_t$ , but this is handled differently in RENORM

# FACTORIZATION BREAKING IN SOFT DIFFRACTION

→ diffractive x-section suppressed relative to Regge prediction as  $\sqrt{s}$  increases



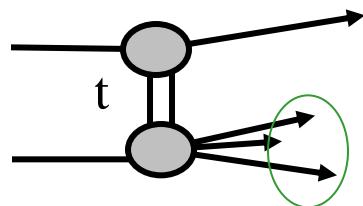
Factor of ~8 (~5)  
suppression at  
 $\sqrt{s} = 1800$  (540) GeV

**RENORMALIZATION**

Interpret flux as gap formation probability  
that saturates when it reaches unity

# Single diffraction renormalized - 1

KG → CORFU-2001: <http://arxiv.org/abs/hep-ph/0203141>



2 independent variables:  $t, \Delta y$

color factor

$$\kappa = \frac{g_{IP-IP-IP}(t)}{\beta_{IP-p-p}(0)} \approx 0.17$$

$$\frac{d^2\sigma}{dt d\Delta y} = C \cdot F_p^2(t) \cdot \underbrace{\left\{ e^{(\varepsilon + \alpha' t)\Delta y} \right\}^2}_{\text{gap probability}} \cdot \kappa \cdot \underbrace{\left\{ \sigma_o e^{\varepsilon \Delta y'} \right\}}_{\text{subenergy x-section}}$$

gap probability

subenergy x-section



Gap probability → (re)normalize to unity

# Single diffraction renormalized - 2

color factor

$$\kappa = \frac{g_{IP-IP-IP}(t)}{\beta_{IP-p-p}(0)} \approx 0.17$$

Experimentally:

KG&JM, PRD 59 (114017) 1999

$$\kappa = \frac{g_{IP-IP-IP}}{\beta_{IP-p}} = 0.17 \pm 0.02, \quad \varepsilon = 0.104$$

QCD:  $\kappa = f_g \times \frac{1}{N_c^2 - 1} + f_q \times \frac{1}{N_c} \xrightarrow{Q^2 = 1} \approx 0.75 \times \frac{1}{8} + 0.25 \times \frac{1}{3} = 0.18$

# Single diffraction renormalized - 3

$$\frac{d^2\sigma_{sd}(s, M^2, t)}{dM^2dt} = \left[ \frac{\sigma_o}{16\pi} \sigma_{IP}^\circ \right] \frac{s^{2\epsilon}}{N(s, s_o)} \frac{e^{bt}}{(M^2)^{1+\epsilon}}$$

$$b = b_0 + 2\alpha' \ln \frac{s}{M^2} \quad s_o^{\text{CMG}} = (3.7 \pm 1.5) \text{ GeV}^2$$

$$N(s, s_o) \equiv \int_{\xi_{\min}}^{\xi_{\max}} d\xi \int_{t=0}^{-\infty} dt f_{IP/p}(\xi, t) \xrightarrow{s \rightarrow \infty} s_o^\epsilon \frac{s^{2\epsilon}}{\ln s}$$

$$\frac{d^2\sigma_{sd}(s, M^2, t)}{dM^2dt} \xrightarrow{s \rightarrow \infty} \sim \ln s \frac{e^{bt}}{(M^2)^{1+\epsilon}}$$

$$\sigma_{sd} \xrightarrow{s \rightarrow \infty} \sim \frac{\ln s}{b \rightarrow \ln s} \Rightarrow const$$

set to unity  
→ determines  $s_o$

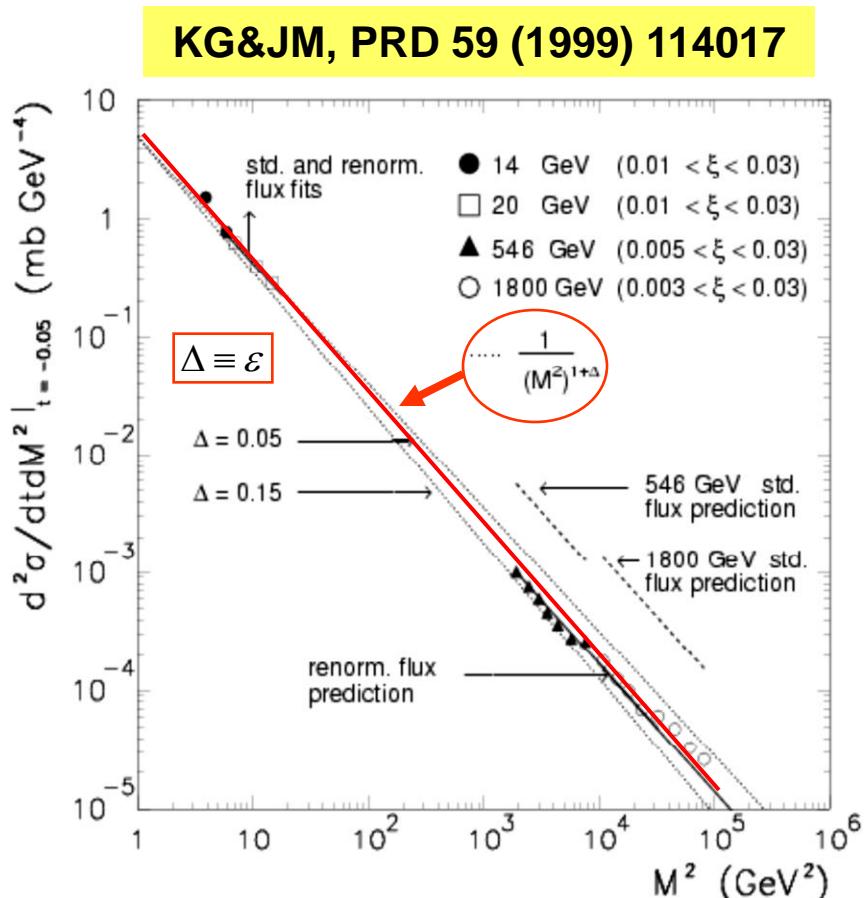
# $M^2$ distribution: data

→  $d\sigma/dM^2|_{t=-0.05} \sim$  independent of  $s$  over 6 orders of magnitude!

Regge

$$\frac{d\sigma}{dM^2} \propto \frac{S^{2\epsilon}}{(M^2)^{1+\epsilon}}$$

Independent of  $S$  over 6  
orders of magnitude in  $M^2$   
→  $M^2$  scaling



→ factorization breaks down to ensure  $M^2$  scaling

# Scale $s_0$ and $PPP$ coupling

Pomeron flux: interpret as gap probability

→ set to unity: determines  $g_{PPP}$  and  $s_0$

KG, PLB 358 (1995) 379

$$\frac{d^2\sigma_{SD}}{dt d\xi} = f_{IP/p}(t, \xi) \sigma_{IP/p}(s\xi)$$

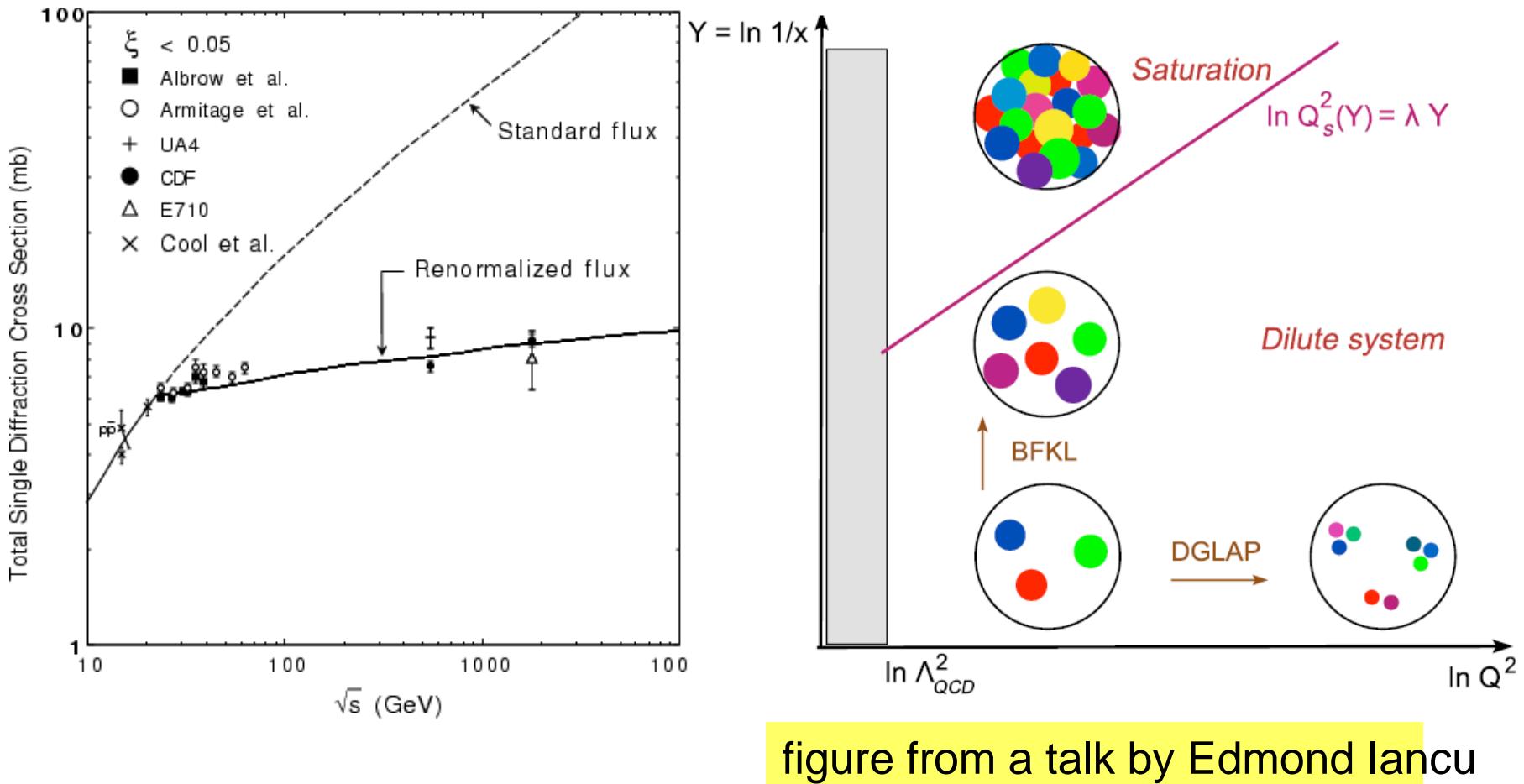
$\downarrow s_0^\varepsilon$

$\uparrow s_0^{-\varepsilon/2} \cdot g_{PPP}(t)$

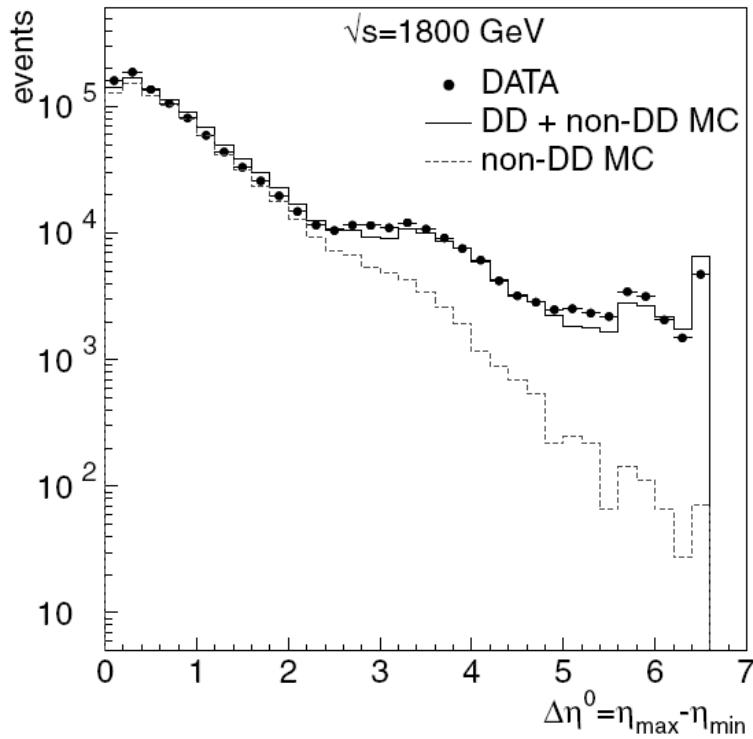
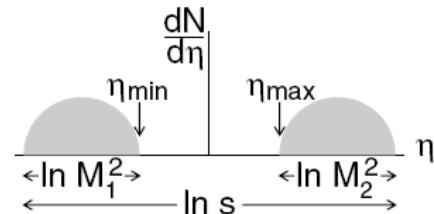
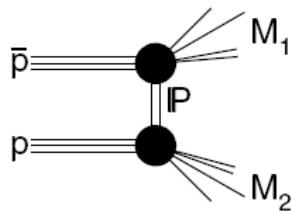
Pomeron-proton x-section

- Two free parameters:  $s_0$  and  $g_{PPP}$
- Obtain product  $g_{PPP} \cdot s_0^{\varepsilon/2}$  from  $\sigma_{SD}$
- Renormalized Pomeron flux determines  $s_0$
- Get unique solution for  $g_{PPP}$

# Saturation at low $Q^2$ and small- $x$



# DD at CDF

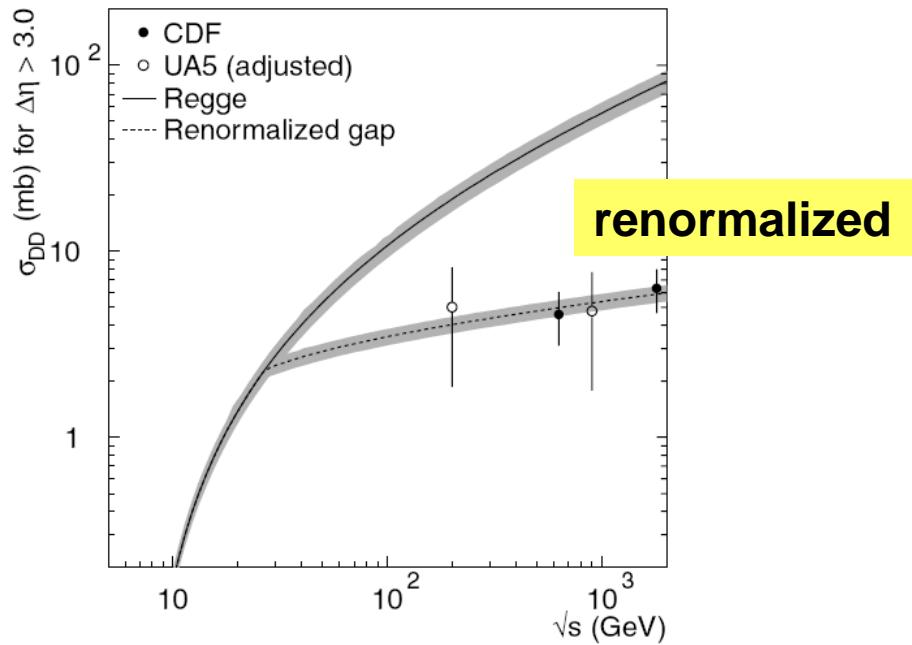


$$\frac{d^3\sigma_{\text{DD}}}{dt dM_1^2 dM_2^2} = \frac{d^2\sigma_{\text{SD}}}{dt dM_1^2} \frac{d^2\sigma_{\text{SD}}}{dt dM_2^2} \Big/ \frac{d\sigma_{el}}{dt}$$

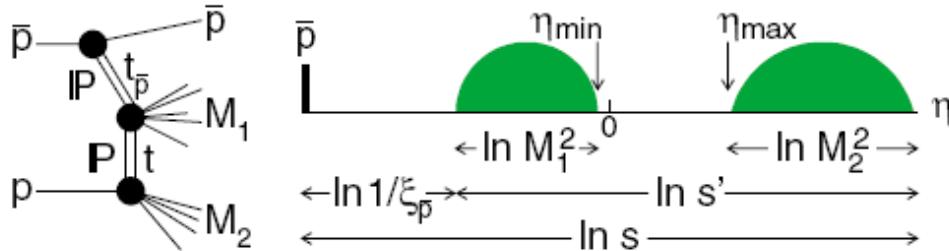
$$= \frac{[\kappa \beta_1(0) \beta_2(0)]^2}{16\pi} \frac{s^{2\epsilon} e^{b_{\text{DD}} t}}{(M_1^2 M_2^2)^{1+2\epsilon}}$$

$$\frac{d^3\sigma_{\text{DD}}}{dt d\Delta\eta d\eta_c} = \left[ \frac{\kappa \beta^2(0)}{16\pi} e^{2[\alpha(t)-1]\Delta\eta} \right] \left[ \kappa \beta^2(0) \left( \frac{s'}{s_0} \right)^\epsilon \right]$$

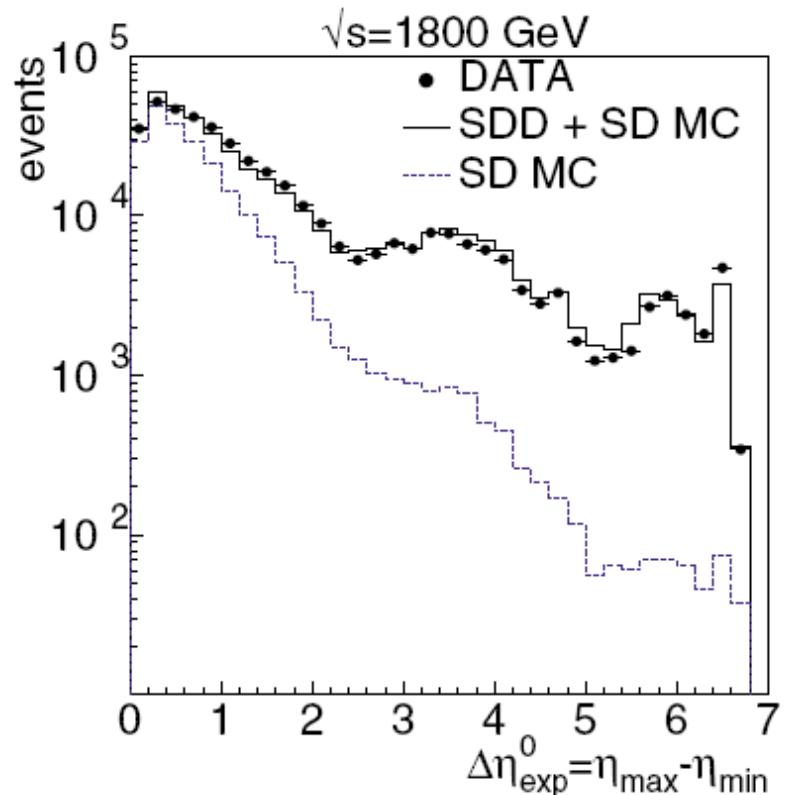
gap probability      x-section



# SDD at CDF

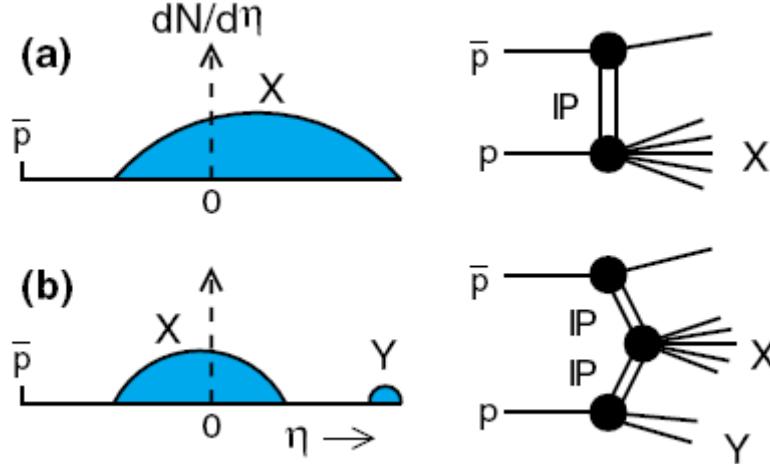


- Excellent agreement between data and MBR (MinBiasRockefeller) MC

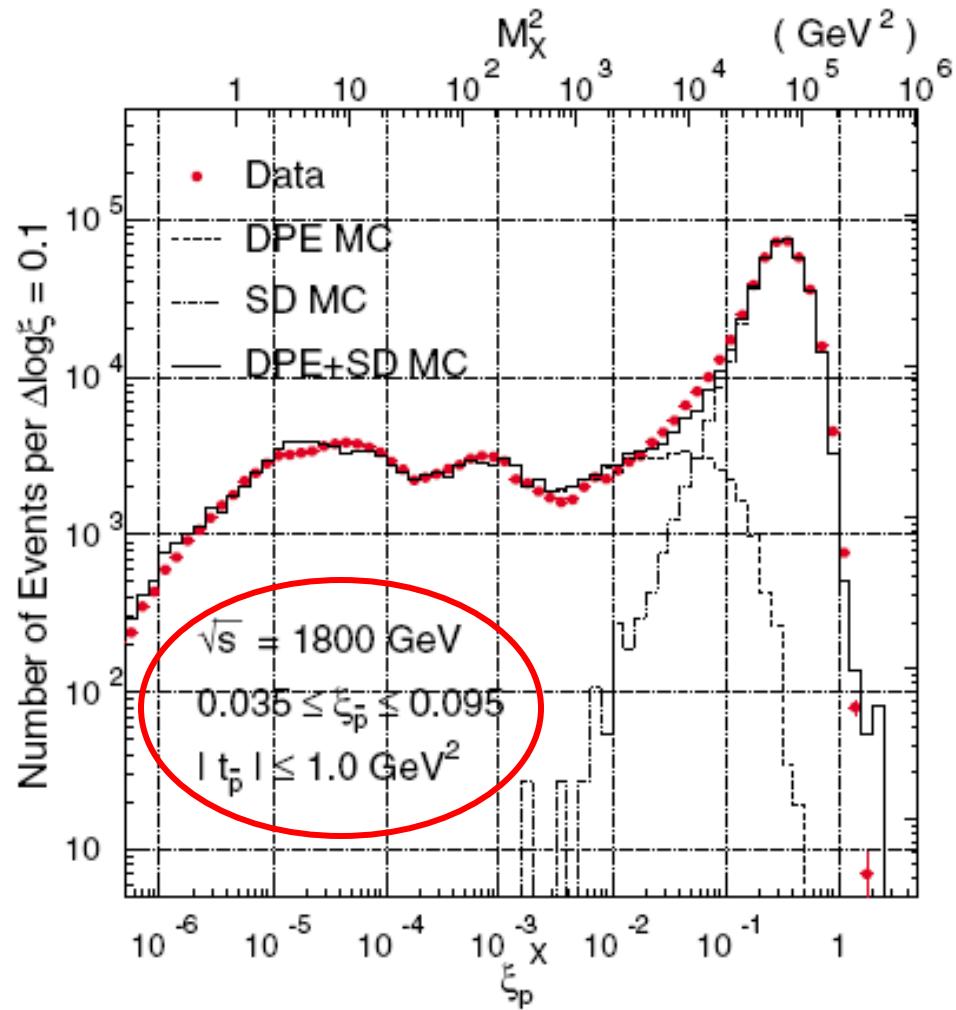


$$\frac{d^5\sigma}{dt_{\bar{p}} dt d\xi_{\bar{p}} d\Delta\eta d\eta_c} = \left[ \frac{\beta(t)}{4\sqrt{\pi}} e^{[\alpha(t_{\bar{p}})-1]\ln(1/\xi)} \right]^2 \times \kappa \left\{ \kappa \left[ \frac{\beta(0)}{4\sqrt{\pi}} e^{[\alpha(t)-1]\Delta\eta} \right]^2 \kappa \left[ \beta^2(0) \left( \frac{s''}{s_o} \right)^{\epsilon} \right] \right\}$$

# CD/DPE at CDF



- Excellent agreement between data and MBR
- low and high masses are correctly implemented



# Diffractive x-sections

$$\begin{aligned}
 \frac{d^2\sigma_{SD}}{dt d\Delta y} &= \frac{1}{N_{\text{gap}}(s)} \left[ \frac{\beta^2(t)}{16\pi} e^{2[\alpha(t)-1]\Delta y} \right] \cdot \left\{ \kappa \beta^2(0) \left( \frac{s'}{s_0} \right)^\epsilon \right\}, \\
 \frac{d^3\sigma_{DD}}{dt d\Delta y dy_0} &= \frac{1}{N_{\text{gap}}(s)} \left[ \frac{\kappa \beta^2(0)}{16\pi} e^{2[\alpha(t)-1]\Delta y} \right] \cdot \left\{ \kappa \beta^2(0) \left( \frac{s'}{s_0} \right)^\epsilon \right\}, \\
 \frac{d^4\sigma_{DPE}}{dt_1 dt_2 d\Delta y dy_c} &= \frac{1}{N_{\text{gap}}(s)} \left[ \prod_i \left[ \frac{\beta^2(t_i)}{16\pi} e^{2[\alpha(t_i)-1]\Delta y_i} \right] \right] \cdot \kappa \left\{ \kappa \beta^2(0) \left( \frac{s'}{s_0} \right)^\epsilon \right\}
 \end{aligned}$$

$$\beta^2(t) = \beta^2(0) F^2(t)$$

$$F^2(t) = \left[ \frac{4m_p^2 - 2.8t}{4m_p^2 - t} \left( \frac{1}{1 - \frac{t}{0.71}} \right)^2 \right]^2 \approx a_1 e^{b_1 t} + a_2 e^{b_2 t}$$

$\alpha_1=0.9$ ,  $\alpha_2=0.1$ ,  $b_1=4.6 \text{ GeV}^{-2}$ ,  $b_2=0.6 \text{ GeV}^{-2}$ ,  $s'=s e^{-\Delta y}$ ,  $\kappa=0.17$ ,  
 $\kappa \beta^2(0)=\sigma_0$ ,  $s_0=1 \text{ GeV}^2$ ,  $\sigma_0=2.82 \text{ mb or } 7.25 \text{ GeV}^{-2}$

# Total, elastic, and inelastic x-sections

$$\sigma_{\text{ND}} = (\sigma_{\text{tot}} - \sigma_{\text{el}}) - (2\sigma_{\text{SD}} + \sigma_{\text{DD}} + \sigma_{\text{CD}})$$

CMG

R. J. M. Covolan, K. Goulianatos, J. Montanha, Phys. Lett. B 389, 176 (1996)

$$\sigma_{\text{tot}}^{p\pm p} = \begin{cases} 16.79s^{0.104} + 60.81s^{-0.32} \mp 31.68s^{-0.54} & \text{for } \sqrt{s} < 1.8 \\ \sigma_{\text{tot}}^{\text{CDF}} + \frac{\pi}{s_0} \left[ \left( \ln \frac{s}{s_F} \right)^2 - \left( \ln \frac{s^{\text{CDF}}}{s_F} \right)^2 \right] & \text{for } \sqrt{s} \geq 1.8 \end{cases}$$

KG Moriond 2011, arXiv:1105.1916

$$\sqrt{s^{\text{CDF}}} = 1.8 \text{ TeV}, \sigma_{\text{tot}}^{\text{CDF}} = 80.03 \pm 2.24 \text{ mb}$$

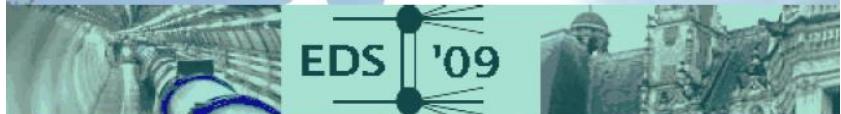
$$\sqrt{s_F} = 22 \text{ GeV} \quad s_0 = 3.7 \pm 1.5 \text{ GeV}^2$$

$\sigma_{\text{el}}^{p\pm p} = \sigma_{\text{tot}} \times (\sigma_{\text{el}}/\sigma_{\text{tot}})$ , with  $\sigma_{\text{el}}/\sigma_{\text{tot}}$  from CMG  
small extrapol. from 1.8 to 7 and up to 50 TeV )

# Diffractive and Total pp Cross Sections at LHC



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- Use the Froissart formula as a *saturated* cross section

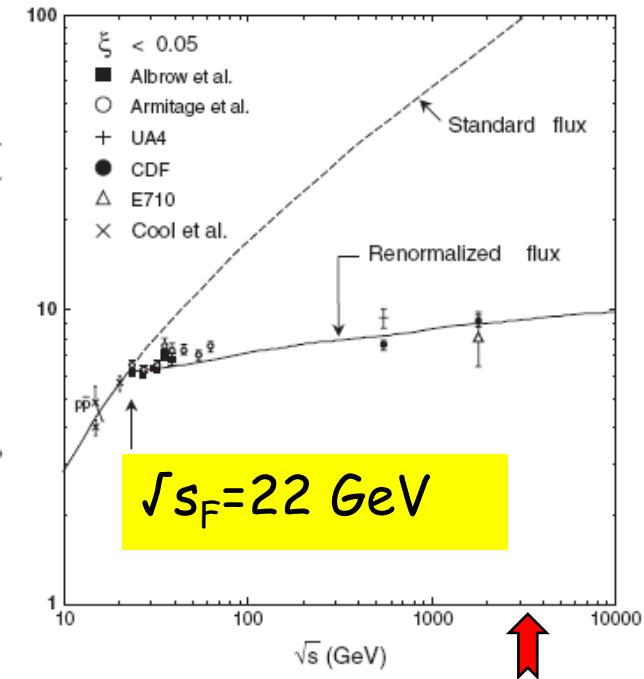
$$\sigma_t(s > s_F) = \sigma_t(s_F) + \frac{\pi}{m^2} \cdot \ln^2 \frac{s}{s_F}$$

- This formula should be valid above the *knee* in  $\sigma_{sd}$  vs.  $\sqrt{s}$  at  $\sqrt{s}_F = 22$  GeV (Fig. 1) and therefore valid at  $\sqrt{s} = 1800$  GeV.
- Use  $m^2 = s_o$  in the Froissart formula multiplied by  $1/0.389$  to convert it to  $\text{mb}^{-1}$ .
- Note that contributions from Reggeon exchanges at  $\sqrt{s} = 1800$  GeV are negligible, as can be verified from the global fit of Ref. [7].
- Obtain the total cross section at the LHC:

$$\sigma_t^{\text{LHC}} = \sigma_t^{\text{CDF}} + \frac{\pi}{s_o} \cdot \left( \ln^2 \frac{s^{\text{LHC}}}{s_F} - \ln^2 \frac{s^{\text{CDF}}}{s_F} \right)$$

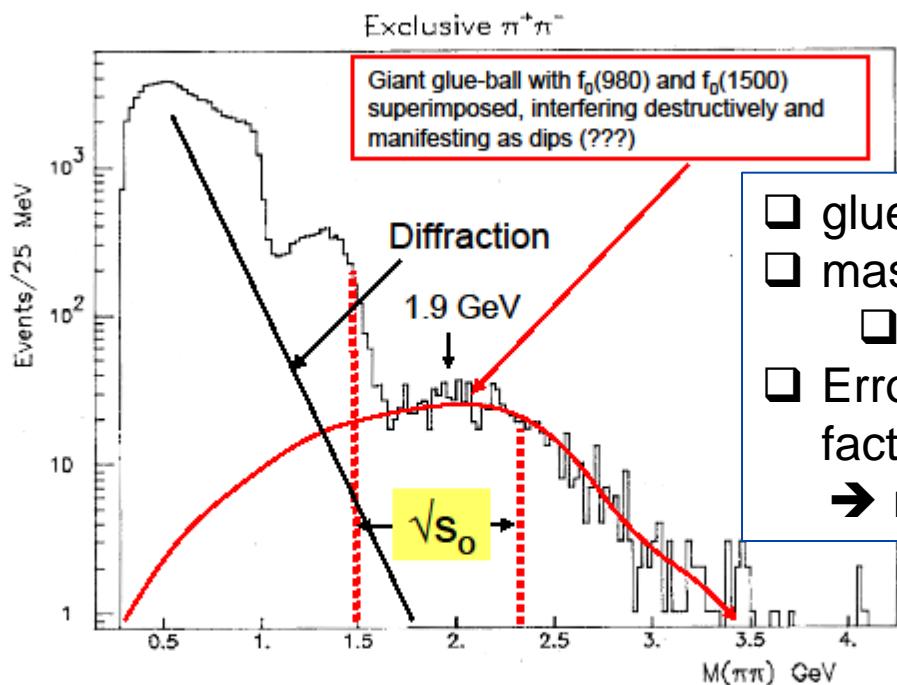
**98 ± 8 mb at 7 TeV**  
**109 ± 12 mb at 14 TeV**

Main error  
from  $s_0$



# Reduce the uncertainty in $s_0$

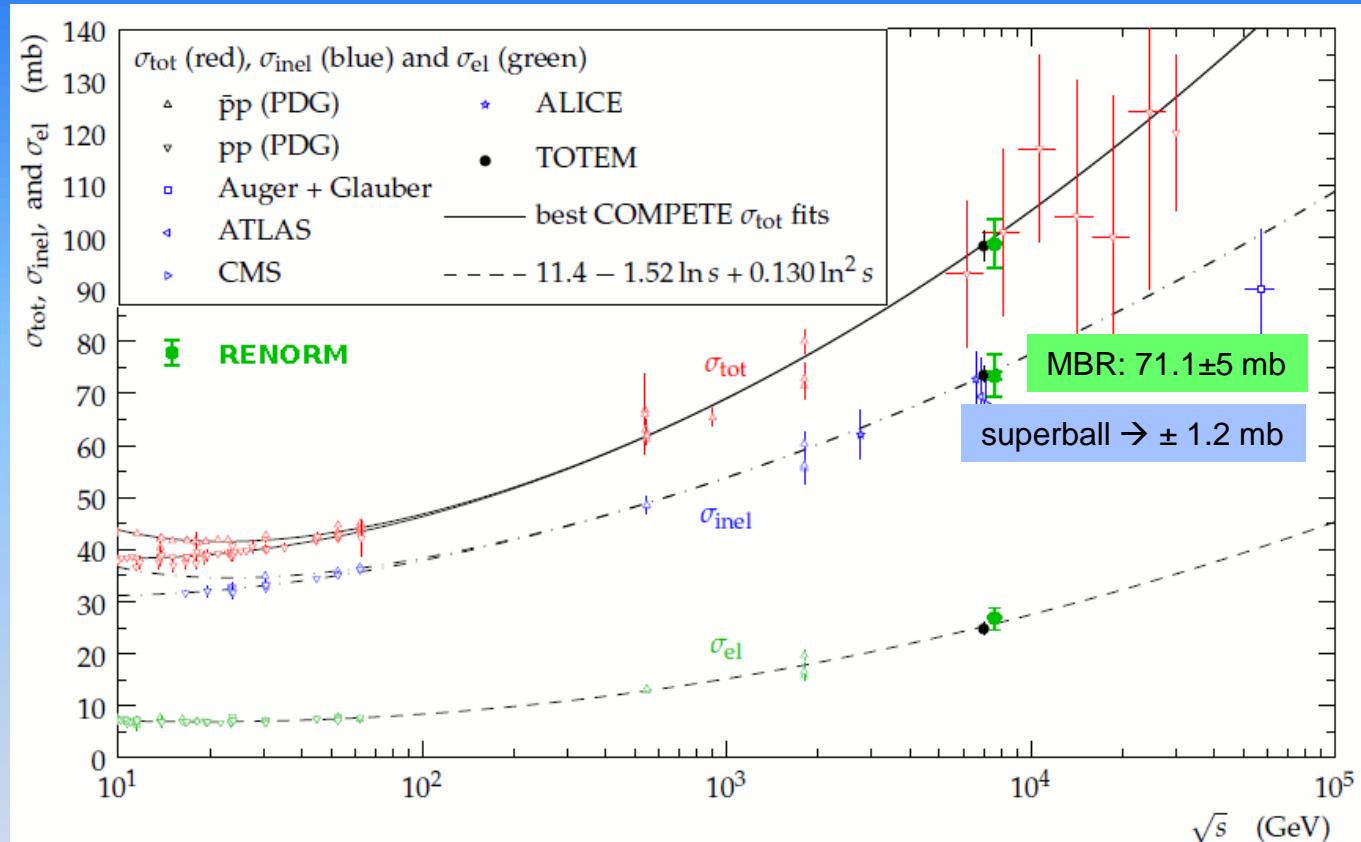
## Saturation glueball?



- glue-ball-like object → “superball”
- mass → 1.9 GeV →  $m_s^2 = 3.7 \text{ GeV}$
- agrees with RENORM  $s_0 = 3.7$
- Error in  $s_0$  can be reduced by factor ~4 from a fit to these data!  
→ reduces error in  $\sigma_t$ .

Figure 8:  $M_{\pi^+\pi^-}$  spectrum in *DIF-E* at the ISR (Axial Field Spectrometer, RS07 [97, 98]). Figure from Ref. [98]. **See M.G.Albrow, T.D. Goughlin, J.R. Forshaw, hep-ph>arXiv:1006.1289**

# TOTEM vs PYTHIA8-MBR



$$\sigma_{\text{inrl}}^{7 \text{ TeV}} = 72.9 \pm 1.5 \text{ mb}$$

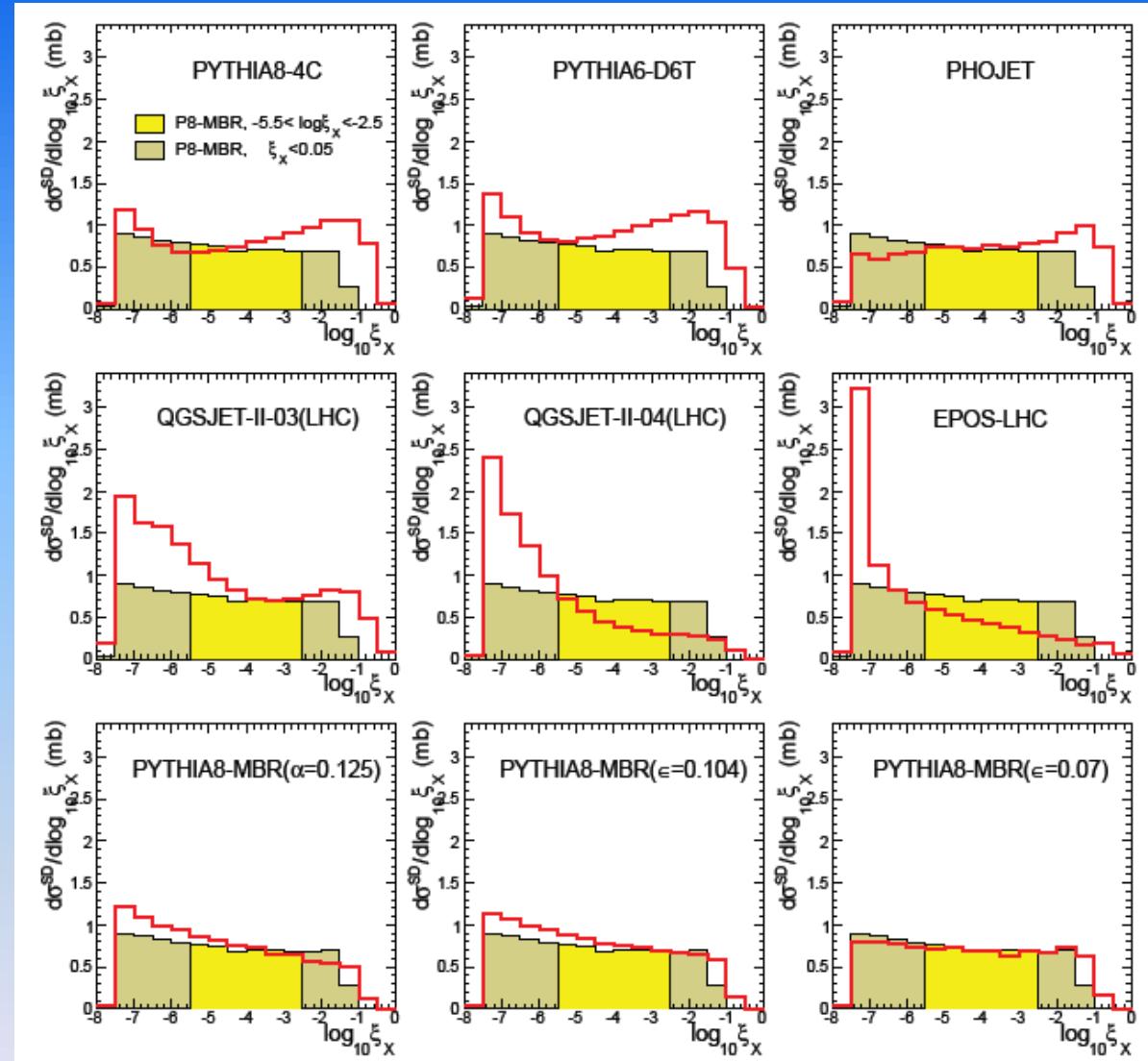
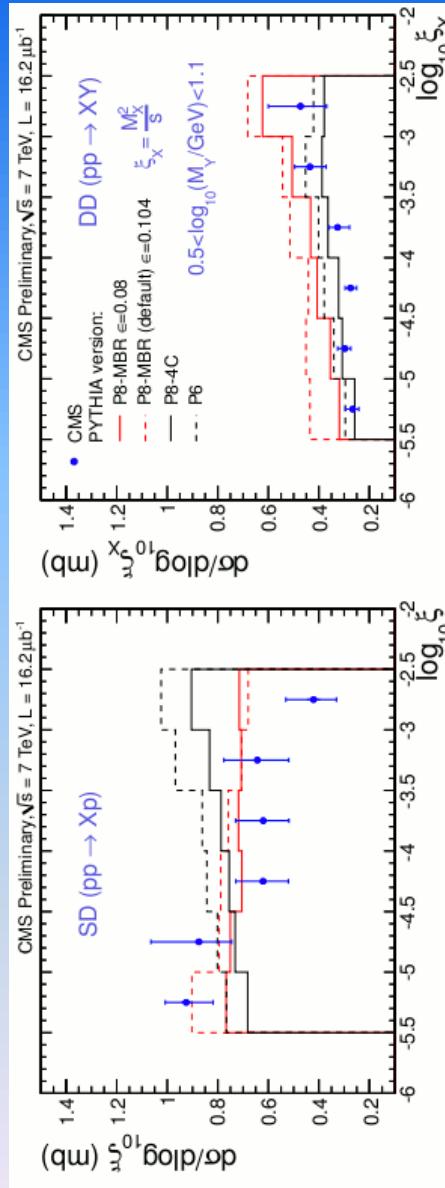
$$\sigma_{\text{inrl}}^{8 \text{ TeV}} = 74.7 \pm 1.7 \text{ mb}$$

*TOTEM, G. Latino talk at MPI@LHC, CERN 2012*

RENNORM:  $71.1 \pm 1.2$  mb

RENNORM:  $72.3 \pm 1.2$  mb

# CMS data best described by PYTHIA8-MBR



Central yellow-filled box is the data region (see left figure)

# $p_T$ distr's of MCs vs Pythia8 tuned to MBR

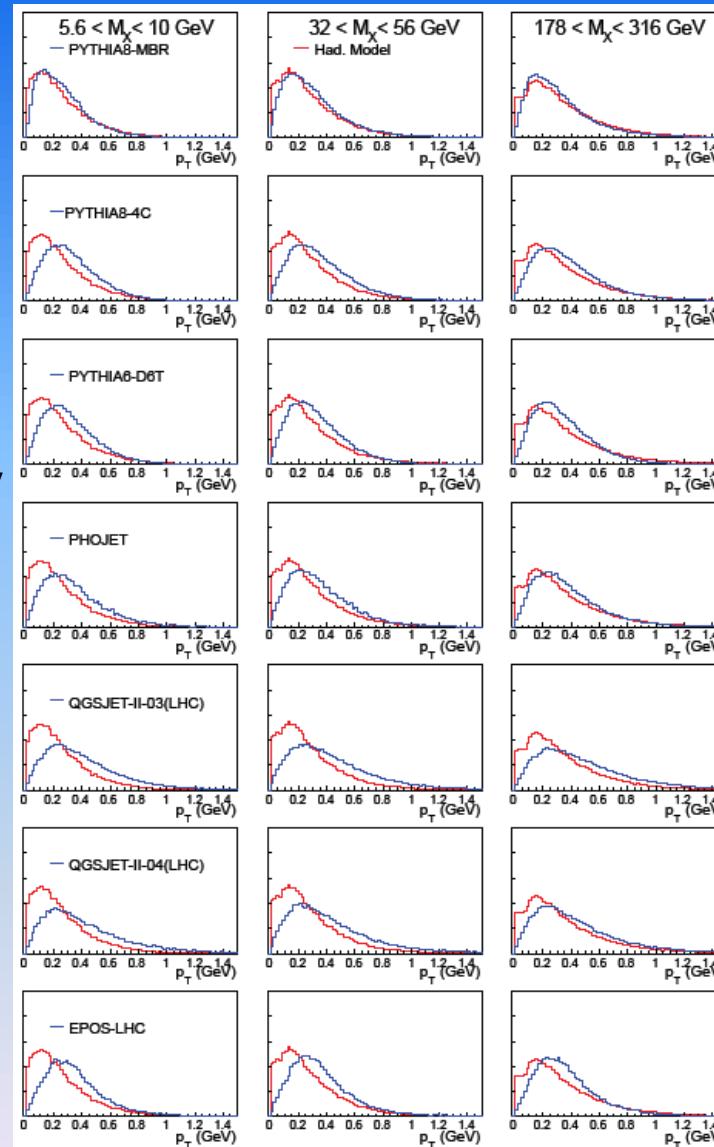
## □ COLUMNS

### Mass Regions

- Low  $5.5 < M_X < 10 \text{ GeV}$
- Med.  $32 < M_X < 56 \text{ GeV}$
- High  $176 < M_X < 316 \text{ GeV}$

## □ CONCLUSION

- PYTHIA8-MBR agrees best with reference model and can be trusted to be used in extrapolating to the unmeasured regions.



← Pythia8 tuned to MBR

## □ ROWS

### MC Models

- PYTHIA8-MBR
- PYTHIA8-4C
- PYTHIA8-D6C
- PHOJET
- QGSJET-II-03(LHC)
- QGSJET-04(LHC)
- EPOS-LHC

# Charged mult's vs MC model – 3 mass regions

**Pythia8 parameters tuned to reproduce  
multiplicities of modified gamma distribution  
KG, PLB 193, 151 (1987)**

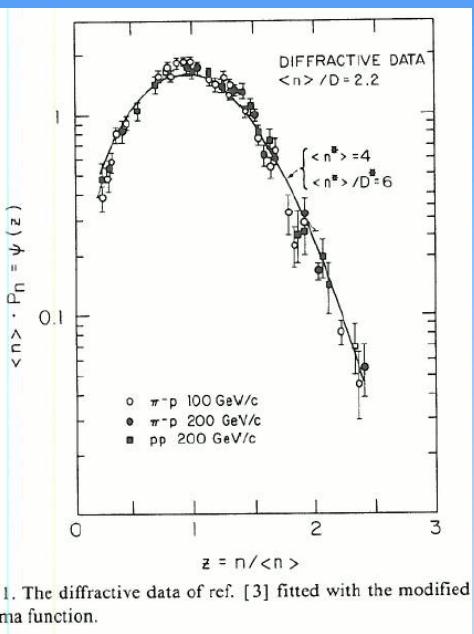


Fig. 1. The diffractive data of ref. [3] fitted with the modified gamma function.

## Mass Regions

- Low  $5.5 < M_X < 10 \text{ GeV}$
- Med.  $32 < M_X < 56 \text{ GeV}$
- High  $176 < M_X < 316 \text{ GeV}$

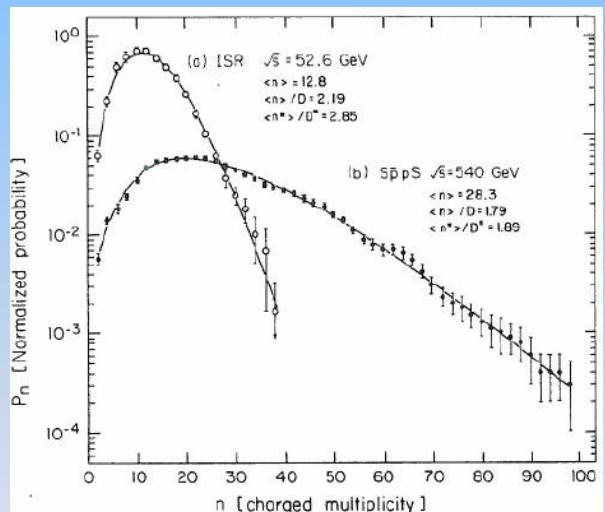
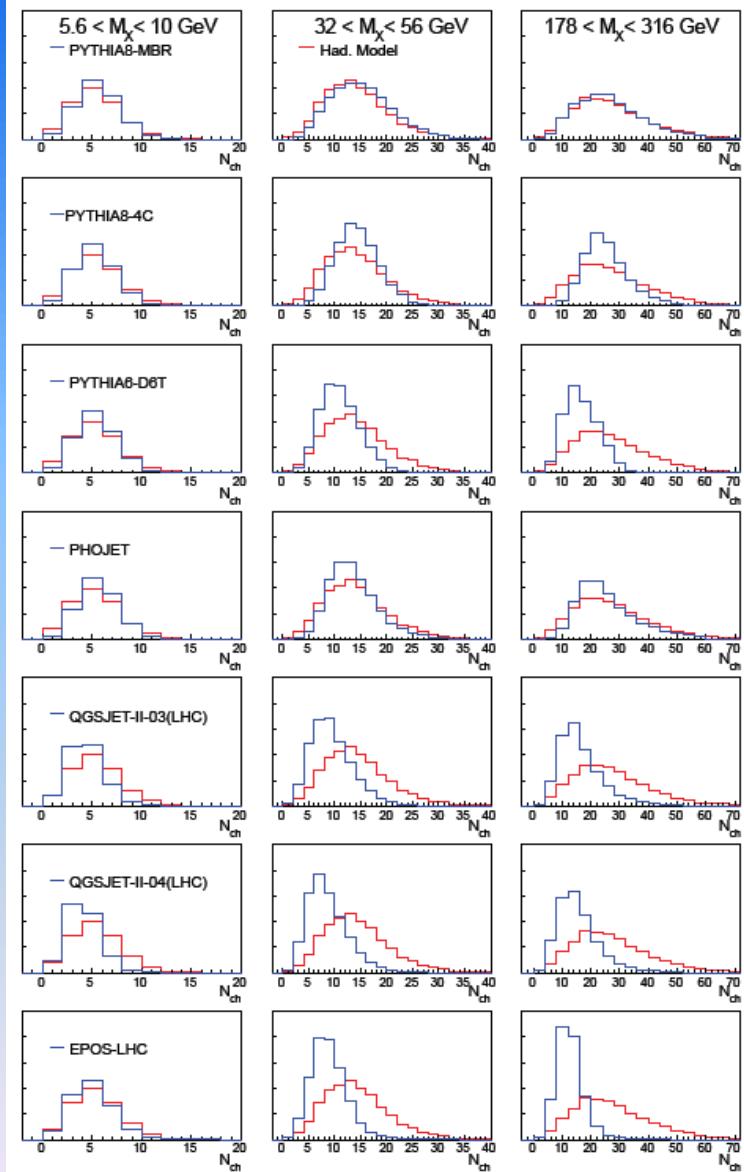


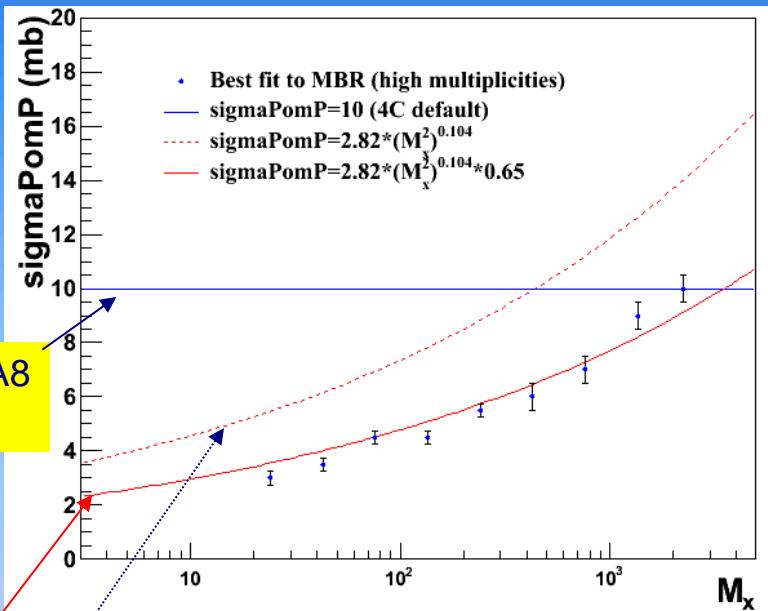
Fig. 2. Full phase space inelastic non-single-diffractive data fitted with the modified gamma function: (a) ISR data [5] at  $\sqrt{s}=52.6 \text{ GeV}$  and (b) collider data [7] at  $\sqrt{s}=540 \text{ GeV}$ .



# Pythia8-MBR hadronization tune

Diffraction: tune SigmaPomP

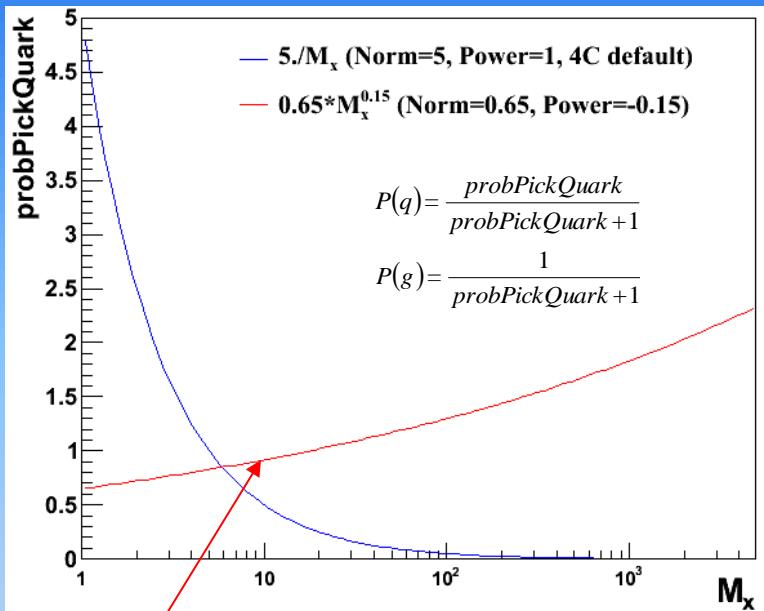
$$n_{ave} = \frac{\sigma_{QCD}}{\sigma_{IPp}}$$



$\sigma^{Pp}(s)$  expected from Regge phenomenology for  $s_0=1$  GeV $^2$  and DL t-dependence.

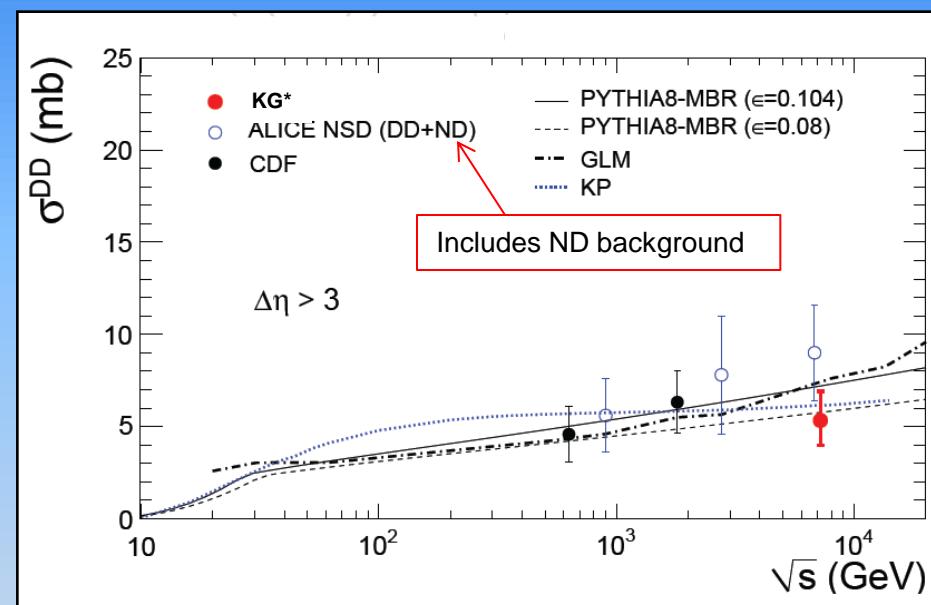
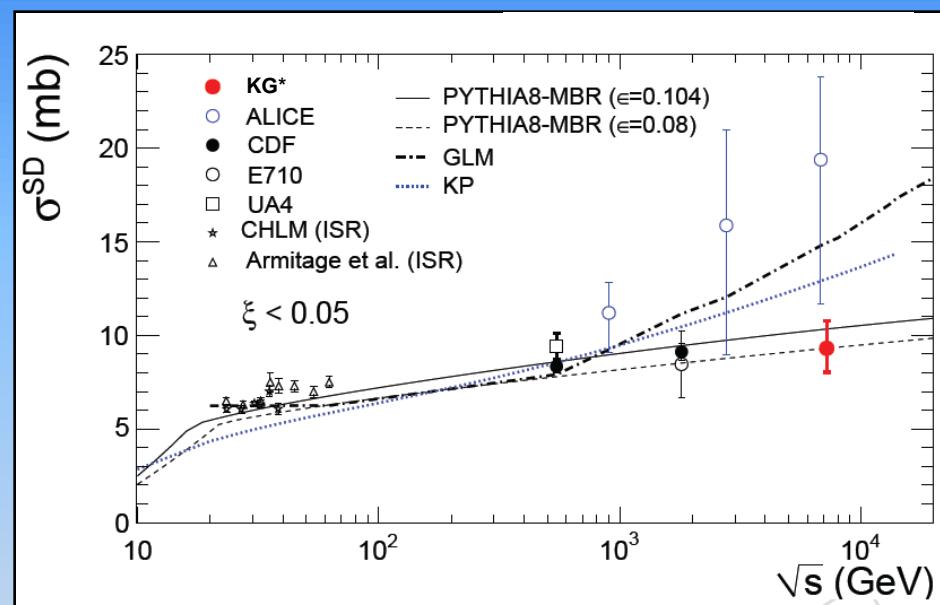
**Red line:** best fit to multiplicity distributions.  
(in bins of  $M_x$ , fits to higher tails only, default pT spectra)

Diffraction: QuarkNorm/Power parameter



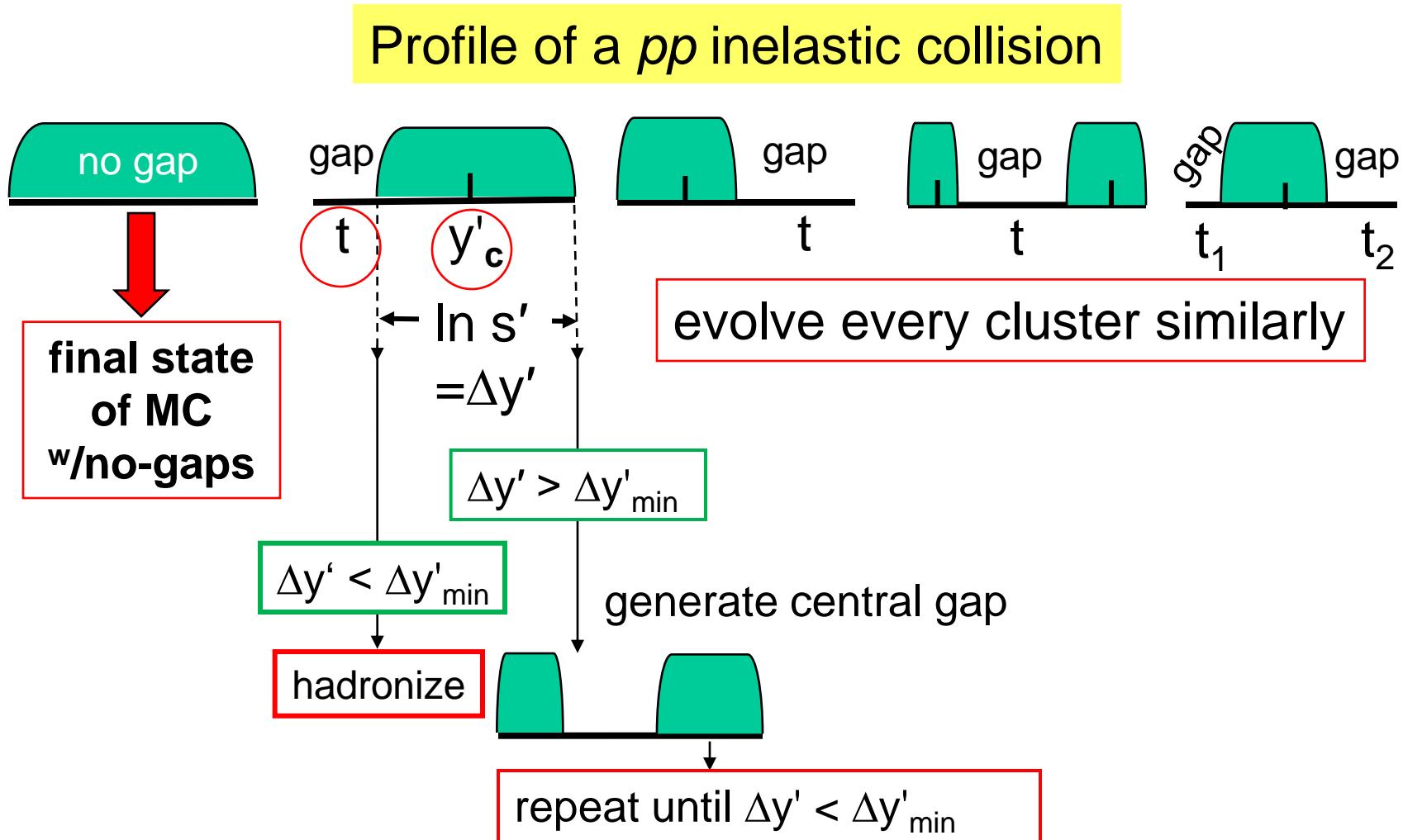
good description of low multiplicity tails

# SD and DD x-sections vs theory



❑ KG\*: after extrapolation into low  $\xi$  from the measured CMS data using MBR model

# Monte Carlo algorithm - nesting



# SUMMARY

- Introduction
  - Diffractive cross sections:
    - basic: SD1,SD2, DD, CD (DPE)
    - combined: multigap x-sections
    - ND → no diffractive gaps:
      - ❖ this is the only final state to be tuned
  - Monte Carlo strategy for the LHC – “nesting”
- } derived from ND and QCD color factors

*Thank you for your attention*