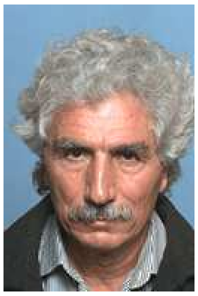
PHENOMENOLOGY OF DIFFRACTIVE PARTON DENSITIES [a]

K. GOULIANOS
The Rockefeller University
1230 York Avenue, New York, NY 10021, USA



The diffractive structure function of the proton is derived from the non-diffractive using a phenomenological model motivated by a scaling behavior observed in soft single diffraction cross sections. Predictions for diffractive deep inelastic scattering are compared with data.

The inclusive and diffractive deep inelastic scattering (DIS) cross sections are proportional to the corresponding F_2 structure functions of the proton,

$$\text{Inclusive DIS} \quad \frac{d^2\sigma}{dxdQ^2} \propto \frac{F_2^h(x,Q^2)}{x}$$

$$\text{Diffractive DIS} \quad \frac{d^3\sigma}{d\xi dxdQ^2} \propto \frac{F_2^{D(3)}(\xi,x,Q^2)}{x}$$

where $\xi = x_{I\!P}$ is the fractional momentum loss of the proton and the superscripts h and $D(3)$ indicate, respectively, a *hard* structure function (at scale Q^2) and a 3-variable diffractive structure function (integrated over t). The latter depends not only on the hard scale Q^2, but also on the *soft* scale, $\langle p_T \rangle \sim 1$ GeV, which is the relevant scale for the formation of the diffractive rapidity gap.

The only marker of the rapidity gap $\Delta\eta \approx \ln\frac{1}{\xi}$ is the variable ξ. We therefore postulate that the rapidity gap probability is proportional to the *soft* parton density at ξ and write the DDIS (diffractive DIS) cross section as

$$\frac{d^3\sigma}{d\xi dxdQ^2} \propto \frac{F_2^h(x,Q^2)}{x} \times \frac{F^s(\xi)}{\xi} \times N(\beta, Q^2, \xi_{min})$$

where $F^s(\xi)/\xi$ is the parton density at the soft scale and $N(\beta, Q^2, \xi_{min})$ is a normalization factor introduced to guarantee that the rapidity gap probability at fixed $\beta \equiv x/\xi$ integrated over all

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available ξ-space, and therefore depending on $\xi_{min} = \frac{x_{min}}{\beta} = \frac{Q^2}{\beta s}$, saturates at unity. This saturation, which prevents unitarity from being violated due to the rapid rise of the structure function with decreasing x, is similar to the saturation caused by the "screening corrections" applicable to the low Q^2 region. The dependence of $N(\beta, Q^2, \xi_{min})$ on $\xi_{min} = Q^2/\beta s$ results in a breakdown of factorization. At high Q^2, where the gap probability is not saturated, factorization should hold [1].

To evaluate $N(\beta, Q^2, \xi_{min})$ at fixed β we first change variables $x \to \beta$ and write the DDIS differential cross section as:

$$\frac{d^3\sigma}{d\xi d\beta dQ^2} \propto \frac{1}{\beta}\left[F_2^h(\beta\xi, Q^2) \times \frac{F_2^s(\xi)}{\xi} \times N(\beta, Q^2, \xi_{min})\right]$$

The term in the brackets represents the DDIS structure function $F_2^{D(3)}(\xi, \beta, Q^2, \xi_{min})$. Note, again, that $F_2^{D(3)}$ depends on ξ_{min} only at low $Q^2/\beta s$ where saturation effects are important.

In the next step, we seek guidance from the scaling behavior of the soft single-diffractive (sd) differential cross section for $pp \to pX$ [1,2],

$$\frac{d\sigma_{sd}}{dM_X^2} \propto \frac{1}{(M_X^2)^{1+\epsilon}} \quad \text{(no } s\text{-dependence} \Rightarrow \text{scaling!)}$$

which in terms of ξ takes the form

$$\frac{d\sigma_{sd}}{d\xi} \propto \underbrace{\frac{1}{s^{2\epsilon}}\frac{1}{\xi^{1+2\epsilon}}}_{\text{gap probability}} \times (s')^{\epsilon}$$

where $s' \equiv M_X^2$ is the s-value of the diffractive sub-system. Noting that ξ is related to the associated rapidity gap by $\Delta\eta = \ln\frac{1}{\xi}$, and that the integral $\int_{s_\circ/s}^{1} \frac{1}{s^{2\epsilon}} \frac{d\xi}{\xi^{1+2\epsilon}}$ is equal to a constant, the above equation may be viewed as representing the product of the total cross section at the sub-system energy, $\sqrt{s'}$, multiplied by a *normalized* rapidity gap probability. In analogy with this experimentally established behavior, we factorize $F_2^{D(3)}(\xi, \beta, Q^2, \xi_{min})$ into $F_2^h(\beta, Q^2)$, the sub-energy DIS cross section, times a normalized gap probability:

$$F_2^{D(3)}(\xi, \beta, Q^2, \xi_{min}) = P_{gap}(\xi, \beta, Q^2, \xi_{min}) \times f_c \times F_2^h(\beta, Q^2)$$

The factor f_c is a "color singlet factor" introduced to account for the fact that the combination of a hard plus a soft exchange must be a color singlet. In DDIS f_c is taken to be equal to $\frac{1}{2}$, since the removal of a quark from the proton on the hard scale must be matched by the removal of an antiquark on the soft scale, limiting the number of (anti)quarks available for the formation of a color singlet to $\frac{1}{2}$ of the total number.

The normalized gap probability is given by

$$P_{gap}(\xi, \beta, Q^2, \xi_{min}) = F_2^h(\beta\xi, Q^2) \times \frac{F^s(\xi)}{\xi} \times N(\beta, Q^2, \xi_{min})$$

where $N(\beta, Q^2, xi_{min})$ is obtained by setting the integral of $P_{gap}(\xi, \beta, Q^2, \xi_{min})$ over ξ to unity:

$$N^{-1}(\beta, \xi_{min}) = \int_{\xi_{min}}^{1} F_2^h(\beta\xi, Q^2) \times \frac{F^s(\xi)}{\xi} d\xi$$

At small x ($\lesssim 0.1$), the structure functions F_2^h and F^s are represented well by the power law expressions [3] $F_2^h(x, Q^2) = A^h/x^{\lambda_h(Q^2)}$ and $F^s(\xi) = A^s/\xi^{\lambda_s}$. Using these forms, and setting $\xi_{min} = Q^2/\beta s$, we obtain

$$N^{-1}(\beta, \xi_{min}) = \frac{A^h}{\beta^{\lambda_h}} \frac{A^s}{\lambda_h + \lambda_s} \left(\frac{\beta s}{Q^2}\right)^{\lambda_h + \lambda_s}$$

which yields

$$F_2^{D(3)}(\xi,\beta,Q^2,\xi_{min}) = \frac{1}{\xi^{1+\lambda_h+\lambda_s}} \times (\lambda_h+\lambda_s)\left(\frac{Q^2}{\beta s}\right)^{\lambda_h+\lambda_s} \times \left[f_c \times \frac{A^h}{\beta^{\lambda_h}}\right] \qquad (1)$$

Since in DDIS x is always smaller than ξ, the above form of $F_2^{D(3)}$, derived for small x, should be valid for all x when ξ is small; it should also be valid for all β, since $\beta = x/\xi$. We therefore expect $F_2^{D(3)}$ to have the following ξ and β dependence at small ξ:

$$F_2^{D(3)}(\xi,\beta,Q^2,\xi_{min})|_{\beta,Q^2} \propto \frac{1}{\xi^{1+n}} \qquad n = \lambda_h(Q^2)+\lambda_s$$

$$F_2^{D(3)}(\xi,\beta,Q^2,\xi_{min})|_{\xi,Q^2} \propto \frac{1}{\beta^m} \qquad m = 2\lambda_h(Q^2)+\lambda_s$$

The HERA (non-diffractive) DIS measurements [3] yield $\lambda_h(Q^2) \approx 0.1 + 0.053 \ln Q^2$. At $Q^2 = 1$ GeV^2, $\lambda_h \to \lambda_s \approx 0.1$, which is in agreement with the value of $\epsilon = \alpha(0) - 1 = 0.104$ [4], where $\alpha(0)$ is the intercept of the Pomeron trajectory at t=0. In the Q^2 range of 10-50 GeV^2, where the DDIS data are concentrated, these measurements yield $\lambda_h \approx 0.3$. Using these values we obtain $n = 0.4$ and $m = 0.7$. We therefore expect

$$\text{Prediction:} \quad F_2^{D(3)} \propto \frac{1}{\xi^{1.4}} \times \frac{1}{\beta^{0.7}}$$

We observe the following features:

Regge factorization

Our prediction exhibits "Regge factorization" between ξ and β, in agreement with HERA results at small values of ξ.

ξ-dependence

In the Regge framework, the ξ-dependence of $F_2^{D(3)}$ is expected to have the "Pomeron flux" form of $1/\xi^{1+n}$ with $n = 2\epsilon = 0.2$, independent of Q^2. In the Q^2 range of 10-50 GeV^2, the HERA experiments find that n is ≈ 0.4 and has a small Q^2 dependence, in agreement with our prediction of $n = \lambda_h(Q^2) + \lambda_s$=0.3+0.1=0.4.

β-dependence

The predicted form $1/\beta^m$ for $F_2^{D(3)}$ holds for small (fixed) ξ and for Q^2 values large enough so that the form $F_2(x,Q^2) = A^h/x^{\lambda_h(Q^2)}$ is approximately valid throughout the entire x region. As there are no data points at strictly fixed ξ, we have selected the following set of five points at $\xi \approx 0.01$ and $Q^2 = 45$ GeV^2 from Ref. [5]:

β	x	$\xi = x/\beta$	$\xi \cdot F_2^{D(3)} \pm stat \pm syst$
0.10	0.00133	0.0133	$0.0384 \pm 0.0066 \pm 0.0030$
0.20	0.00237	0.0118	$0.0406 \pm 0.0061 \pm 0.0026$
0.40	0.00421	0.0105	$0.0215 \pm 0.0046 \pm 0.0016$
0.65	0.00750	0.0115	$0.0240 \pm 0.0054 \pm 0.0026$
0.90	0.00750	0.0083	$0.0088 \pm 0.0041 \pm 0.0005$

Figure 1 shows the above values of $\xi \cdot F_2^{D(3)}(\beta)$ versus β along with our prediction (solid line). The following parameters were used in the calculation of $F_2^{D(3)}$: $\sqrt{s} = 280$ GeV, $\xi = 0.01$, $Q^2 = 45$ GeV^2, $\lambda_s = 0.1$, $\lambda_h = 0.3$, $f_c = 0.5$, and $A^h = 0.2$; the latter was evaluated from $F_2(Q^2 = 50, x = 0.00133) = 1.46$ [6] assuming a $\frac{A^h}{x^{0.3}}$ dependence. The observed agreement between data and prediction, both in shape and normalization, is considered satisfactory, particularly since no free parameters are used in the calculation.

QCD factorization

Equation (1) was derived under the assumption that the rapidity gap probability is saturated

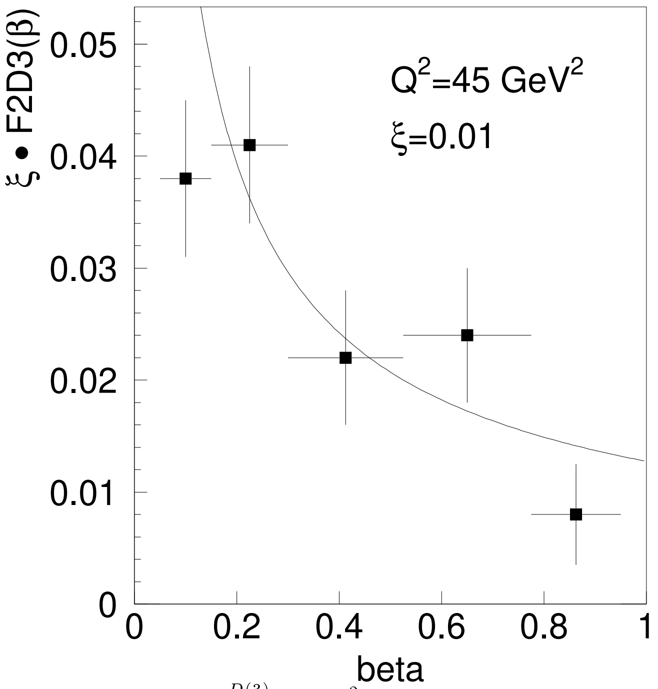

Figure 1: Predicted β dependence of $\xi \times F_2^{D(3)}(\xi, \beta, Q^2, \xi_{min})$ for $\xi = 0.01$ and $Q^2 = 45$ GeV^2 (solid curve) compared with measured values (points) obtained from Ref. [5]

and therefore must be normalized to unity. This normalization introduces the factor $(Q^2/\beta s)^n$, which overtly breaks down factorization. Factorization at high Q^2 may be preserved by replacing $(Q^2/\beta s)^n$ in Eq. (1) by the factor $C \times \left(1 - e^{-\frac{1}{C}(Q^2/\beta s)^n}\right)$, in which the constant $C = \xi_0$, to be determined experimentally, is the value of $\xi_{min} = Q^2/\beta s$ above which the gap probability integrates out to less than unity and therefore should not be renormalized. With this modification, the $F_2^{D(3)}$ becomes

$$F_2^{D(3)}(\xi, \beta, Q^2, Q^2/\beta s) = \frac{C}{\xi^{1+\lambda_h+\lambda_s}} \times (\lambda_h + \lambda_s) \left(1 - e^{-\frac{1}{C}(Q^2/\beta s)^n}\right) \times \left[f_c \times \frac{A^h}{\beta^{\lambda_h}}\right]$$

For small $Q^2/\beta s$ the above equation reverts to Eq. (1), which violates QCD factorization, while for large Q^2, $F_2^{D(3)}$ obeys both Regge and QCD factorization.

References

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