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QCD ASPECTS OF HADRONIC DIFFRACTION *

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Experimental results on soft and hard diffractive processes obtained by the CDF Collaboration in $\bar{p}p$ interactions are examined with emphasis on regularities that point to QCD aspects of hadronic diffraction. Data are interpreted in a phenomenological approach in which diffractive cross sections are related to the underlying inclusive parton distribution functions of the nucleon. In this approach, diffraction appears to be mediated by the exchange of low-x partons subject to color constraints.

Keywords: diffraction; factorization; renormalization.

1. Introduction

Diffractive processes are characterized by large rapidity gaps, defined as regions of (pseudo)rapidity[a] devoid of particles. While rapidity gaps can occur in non-diffractive (ND) interactions by fluctuations in particle multiplicities, the probability for such events is expected by Poisson statistics to be exponentially suppressed as a function of gap width [1]. In contrast, diffractive gaps do not exhibit such a suppression. This aspect of diffraction could be explained if the exchange across the gap were a color singlet quark/gluon object with vacuum quantum numbers, historically referred to as the **Pomeron** [2]. In this paper, we attempt to understand the QCD nature of the Pomeron by examining how results on soft and hard diffractive processes obtained by the CDF Collaboration in $\bar{p}p$ interactions might be related to the underlying inclusive parton distribution functions of the interacting nucleons [3].

2. Soft diffraction

The following soft $\bar{p}p$ processes have been studied by CDF:

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[a]We use *rapidity* and *pseudorapidity* interchangeably; in the kinematic region of interest in this presentation the pseudorapidity of a particle, defined as $\eta = -\ln\tan\frac{\theta}{2}$, where θ is the polar angle, is numerically very close to its rapidity, $y = \frac{1}{2}\frac{E+p_L}{E-p_L}$, where p_L is the longitudinal momentum.

ND	Non-Diffractive	$\bar{p}p \to X$
SD	Single Diffraction [4]	$\bar{p}p \to \bar{p} + \text{gap} + X$
DD	Double Diffraction [5]	$\bar{p}p \to X + \text{gap} + Y$
DPE	Double Pomeron Exchange [6]	$\bar{p}p \to \bar{p} + \text{gap} + X + \text{gap} + p$
SDD	Single $\oplus$ Double Diffraction [7]	$\bar{p}p \to \bar{p} + \text{gap} + X + \text{gap} + Y$

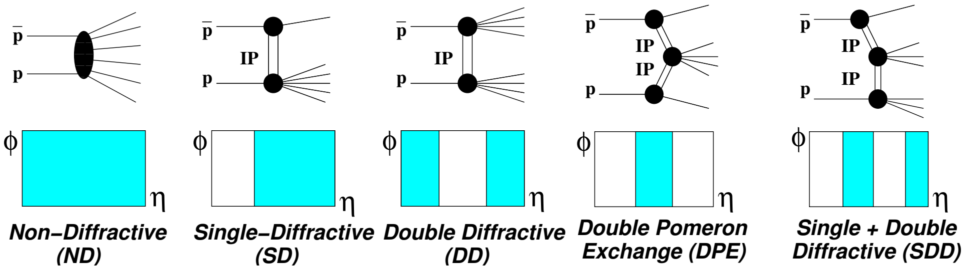

Fig. 1. Diagrams and η-ϕ topologies of soft processes studied by CDF; the shaded areas are regions where particle production occurs and are referred to in this paper as diffractive clusters.

Diffraction has traditionally been treated in Regge theory using factorization. This approach was successful at $\sqrt{s}$ energies below ~ 50 GeV [8], but as the available energies increased to reach $\sqrt{s}$ =1800 GeV at the Tevatron, a suppression as large as $\sim \mathcal{O}(10)$ of the SD cross section was observed [4]. This breakdown of factorization was traced to the energy dependence of $\sigma_{sd}^{tot}(s) \sim s^{2\epsilon}$, which is faster than that of $\sigma^{tot}(s) \sim s^{\epsilon}$, so that at high $\sqrt{s}$ unitarity would have to be violated if factorization held. The s-dependence appears explicitly in the SD differential cross section,

$$\text{Regge theory: } d\sigma_{sd}(s, M^2)/dM^2 \sim s^{2\epsilon}/(M^2)^{1+\epsilon} \qquad (1)$$

2.1. *M^2-scaling and renormalization*

Contrary to expectations from Regge theory based on factorization, the measured SD M^2-distribution does not show any s-dependence over a region of s spanning several orders of magnitude [9]. Thus, it appears that factorization breaks down in such a way as to enforce M^2-scaling. This property is built into the *Renormalization Model* of hadronic diffraction, in which the Regge theory Pomeron flux is renormalized to unity [10]. Below, we present a QCD basis for renormalization and extend the model to central and multigap diffraction [11].

2.2. *QCD basis of renormalization*

The form of the rise of total cross sections at high energies, $\sim s^{\epsilon}$, which in Regge theory requires a Pomeron trajectory with intercept $\alpha(0) = 1 + \epsilon$, is expected in a parton model approach, where cross sections are proportional to the number of available wee partons[12]. In terms of the rapidity region in which there is particle production[b], $\Delta\eta'$, the total pp cross section is given by

$$\sigma_{pp}^{tot} = \sigma_0 \cdot e^{\epsilon\Delta\eta'} \tag{2}$$

Since from the optical theorem $\sigma_{tot} \sim \mathrm{Im}\, \mathrm{f}^{\mathrm{el}}(t=0)$, the full parton model amplitude may be written as

$$\mathrm{Im}\, \mathrm{f}^{\mathrm{el}}(t, \Delta\eta) \sim e^{(\epsilon+\alpha' t)\Delta\eta} \tag{3}$$

where the term $\alpha' t$ is a parameterization of the t-dependence of the amplitude.

Based on this amplitude, the diffractive cross sections of Fig. 1 are expected to have the forms

$$\begin{aligned}
\frac{d^2\sigma_{sd}}{dt\, d\Delta\eta} &= N_{gap}^{-1}(s) \quad F_p(t) \left\{ e^{[\epsilon+\alpha'(t)]\Delta\eta} \right\}^2 \quad \kappa \left[\sigma_0 e^{\epsilon\Delta\eta'} \right] \\
\frac{d^3\sigma_{dd}}{dt\, d\Delta\eta\, d\eta_c} &= N_{gap}^{-1}(s) \quad \left\{ e^{[\epsilon+\alpha'(t)]\Delta\eta} \right\}^2 \quad \kappa \left[\sigma_0 e^{\epsilon(\Sigma_i \Delta\eta_i')} \right] \\
\frac{d^4\sigma_{sdd}}{dt_1\, dt_2\, d\Delta\eta\, d\eta_c} &= N_{gap}^{-1}(s) \ F_p(t)\Pi_i \left\{ e^{[\epsilon+\alpha'(t_i)]\Delta\eta_i} \right\}^2 \ \kappa^2 \left[\sigma_0 e^{\epsilon(\Sigma_i \Delta\eta_i')} \right] \\
\frac{d^4\sigma_{dpe}}{dt_1\, dt_2\, d\Delta\eta\, d\eta_c'} &= N_{gap}^{-1}(s) \ \underbrace{\Pi_i \left\{ F_p(t_i) e^{[\epsilon+\alpha'(t_i)]\Delta\eta_i} \right\}^2}_{\text{gap probability factor}} \ \kappa^2 \underbrace{\left[\sigma_0 e^{\epsilon(\Delta\eta')} \right]}_{\sigma^{tot}(s')}
\end{aligned} \tag{4}$$

where the (re)normalization factor $N_{gap}(s)$ is the integral of the gap probability factor over all phase space in t_i, $\Delta\eta_i$ and the variables η_c and η_c', represent the center of the "floating" (not adjacent to a nucleon) rapidity gap in DD or SDD and the floating cluster in DPE, respectively. In each case, the independent variables are the ones on the left hand side of the equation, but for pedagogical reason we use on the right hand side additional variables, which can be expressed in terms of the ones on the left.

A remarkable property of the above expressions is that they factorize into two terms, one depending on the rapidity region(s) in which there is particle production, and another that depends on the rapidity gap(s). This is due to the exponential dependence on $\Delta\eta$ of the elastic amplitude, which allows non-contiguous regions in rapidity to be added in the exponent. A consequence of this is that the (re)normalization factor is $\sim s^{2\epsilon}$ in all cases, ensuring **universal M^2-scaling**.

These expressions may be understood as follows: (i) the term in square brackets represents the nucleon-nucleon total cross section at the reduced energy defined by $\ln(s'/s_0) = \sum_i \Delta\eta_i'$; (ii) the factors κ, one for each gap, are the color factors required to enable rapidity gap formation; (iii) the gap probability factor is the amplitude squared of the elastic scattering between a diffractively dissociated and a surviving proton, in which case it contains the proton form factor, $F_p(t)$, or between two diffractively dissociated protons. Since the reduced energy cross section is properly normalized, the gap probability term is (re) normalized to unity.

[b]We assume $p_T = 1$ GeV so that $\Delta y' = \Delta\eta'$.

The parameters ϵ and κ in Eqs. (4) have been experimentally found to be[13,9]

$$\begin{aligned} \text{experiment: } \epsilon &\equiv \alpha_{I\!P}(0) - 1 = 0.104 \pm 0.002 \pm 0.01 \text{ (syst)} \\ \kappa &\equiv \frac{g_{I\!P I\!P I\!P}}{\beta_{I\!P p}} = 0.17 \pm 0.02 \text{ (syst)} \end{aligned} \tag{5}$$

where the systematic error assigned to ϵ is a rough estimate by this author based on considering fits made to cross section data by various authors.

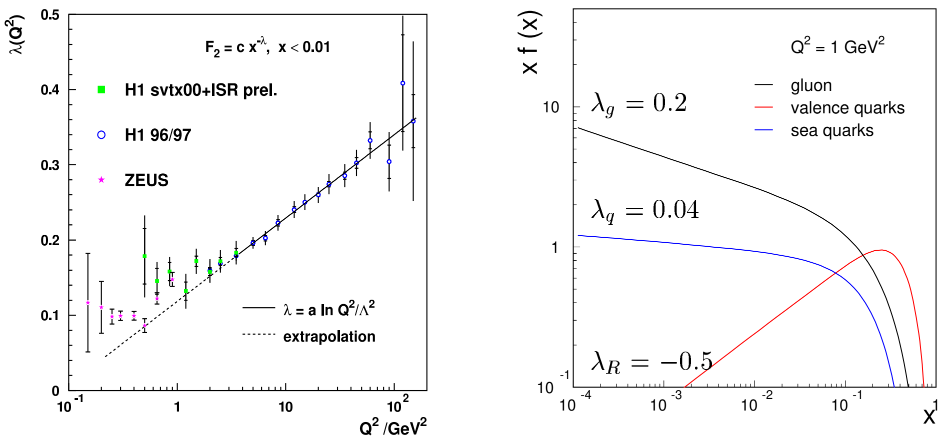

Fig. 2. *(left) The parameter λ versus Q^2 of a fit to the structure function $F_2(x, Q^2) \sim x^{-\lambda}$ in DIS at HERA*[14]*; (right) CTEQ5L nucleon parton distribution functions for $Q^2 = 1$ GeV^2.*

Measurements of parton densities at HERA indicate that partonic structure in the nucleon may exist down to the hadron mass scale of $Q^2 \approx 1$ GeV^2. This is seen in Fig. 2 *left*), where the parameter $\lambda(Q^2)$ of $F_2(x, Q^2) \sim x^{-\lambda}$ decreases linearly with $\ln Q^2$ down to $Q^2 \approx 1$ GeV^2, flattening out and becoming consistent with $\epsilon = 0.1$ only below $Q^2 = 1$ GeV^2. We therefore assume partonic behavior in diffractive interactions and attempt to derive the parameters ϵ and κ from the nucleon parton distribution functions (pdf's) at $Q^2 = 1$ GeV^2, shown in Fig. 2 (*right*) for the CTEQ5L parameterization.

The region of interest to diffraction, $x \leq 0.1$, is dominated by sea gluons and quarks. In this region, a fit of the form $xf(x) \sim x^{-\lambda}$ in Fig. 2 (*right*) yields $\lambda_g \approx 0.2$ and $\lambda_q \approx 0.04$ with relative weights $w_g \approx 0.75$ and $w_q \approx 0.25$ [c]. Noting that the number of wee partons grows as $\int_{1/s}^{1} f(x)dx \sim s^{\lambda}$, the Pomeron intercept may be obtained from the parameters λ_g and λ_q, appropriately weighted by a procedure

[c]For valence quarks $\lambda \approx -0.5$; this is relevant for Reggeon contributions, which are not being considered in this paper.

involving gluon and quark color factors,

$$c_g = \frac{1}{N_c^2 - 1}, \quad c_q = \frac{1}{N_c} \tag{6}$$

Weighting places ϵ in the range $\lambda_q < \epsilon < \lambda_g$, or $0.04 < \epsilon < 0.2$, which covers the experimental value of $\epsilon = 0.104$. The parameter κ is obtained from the g/q color factors and weights:

$$\kappa \approx c_g w_g + c_q w_q = 0.18 \tag{7}$$

This prediction is in remarkably good agreement with $\kappa_{exp} = 0.17 \pm 0.02$.

3. Hard diffraction

Hard diffraction processes are defined as those in which there is a hard partonic scattering in addition to the diffractive rapidity gap signature. Events may have forward, central, or multiple rapidity gaps, as shown in Fig. (3) (*left*) for dijet production in $\bar{p}p$ collisions at the Tevatron, and (*right*) for diffractive deep inelastic scattering (DIS) in *ep* collisions at HERA.

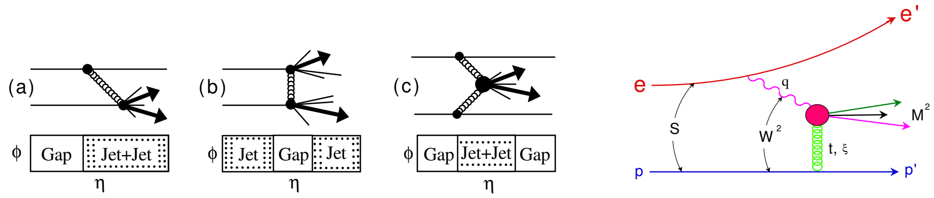

Fig. 3. *(left) Dijet production diagrams and event topologies for $\bar{p}p$ (a) single diffraction (b) double diffraction and (c) double Pomeron exchange; (right) diffractive photon dissociation in ep collisions.*

CDF has measured SD/ND ratios for W, dijet, b-quark and J/ψ production, and also diffractive structure functions from single diffractive and double Pomeron exchange dijet production. The following aspects of the data are of interest:

- All measured SD/ND ratios at $\sqrt{s}$ =1800 GeV are approximately equal ($\sim 1\%$), pointing to a *flavor independent* rapidity gap formation probability.
- The SD/ND structure function ratio varies as $\sim x_{Bj}^{-0.45}$. In contrast, in deep inelastic scattering (DIS) at HERA, a constant ratio is observed [15].
- The SD structure function measured from dijet production at CDF is suppressed by $\sim \mathcal{O}(10)$ relative to expectations from diffractive parton distribution functions (pdf's) measured from diffractive DIS at HERA.
- The Pomeron intercept measured in diffractive DIS at HERA increases with Q^2 and is on average about a factor of two larger than the soft Pomeron intercept.

The above regularities are interpreted below in a QCD framework [3].

3.1. *Hard diffraction in a QCD framework*

In this section we adapt the QCD approach of section 2.2 to diffractive DIS at HERA and diffractive dijet production at the Tevatron:

$$\text{HERA:}\ \ \gamma^* + p \rightarrow p + Jet + X$$
$$\text{Tevatron:}\ \ \bar{p} + p \rightarrow +\text{dijet} + X$$

The hard process may involve several color "emissions" from the surviving proton, comprising a color singlet with vacuum quantum numbers. Two of the emissions have special importance: the one at $x = x_{Bj}$ from the proton's hard pdf at scale Q^2, which is responsible for the hard scattering, and the other at $x = \xi$ (fractional momentum loss of the diffracted nucleon) from the soft pdf at $Q^2 \approx 1$ GeV^2, which neutralizes the exchanged color and forms the rapidity gap. The diffractive structure function should then be the product of the inclusive structure function and the soft parton density at $x = \xi$ [for simplicity, we do not include t dependence],

$$F^D(\xi, x, Q^2) \propto \frac{1}{\xi^{1+\epsilon}} \cdot F(x, Q^2) \sim \frac{1}{\xi^{1+\epsilon}} \cdot \frac{C(Q^2)}{(\beta\xi)^{\lambda(Q^2)}} \Rightarrow \frac{A_{\rm norm}}{\xi^{1+\epsilon+\lambda}} \cdot c_{g,q} \frac{C}{\beta^\lambda} \qquad (8)$$

where $c_{g,q}$ are the color factors shown in Eq. (6), λ is the parameter of a power law fit to the relevant hard structure function in the region $x < 0.1$ (see Fig. 4), and $A_{\rm norm}$ is a normalization factor.

$$x \cdot f(x) \propto x^{-\lambda}$$

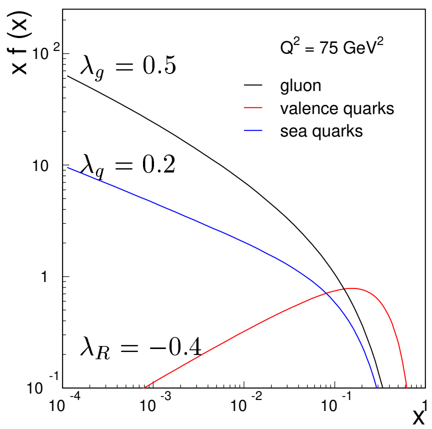

Fig. 4. *CTEQ5L nucleon parton distribution functions for* $Q^2 = 75$ *GeV*^{2}*. The parameters* $\lambda_{g,q,R}$ *are the slopes of the gluon, sea quark, and valence quark distribution ('R' stands for Reggeon) in the region of* $x < 0.1$ *where the power law behavior holds.*

3.2. *Predictions for the diffractive structure function at HERA*

At high Q^2 at HERA, where factorization is expected to hold [10,16], $A_{\rm norm}$ is simply the (constant) normalization factor of the soft pdf. This constant normalization leads to two important predictions:

- The Pomeron intercept in diffractive DIS (DDIS) is Q^2-dependent and equals the average of the soft and hard intercepts:

$$\begin{aligned} \alpha_{I\!P}^{DIS} &= 1 + \lambda(Q^2) \\ \alpha_{I\!P}^{DDIS} &= 1 + \frac{1}{2}\left[\epsilon + \lambda(Q^2)\right] \end{aligned} \tag{9}$$

- The ratio of DDIS to DIS structure functions at a given ξ is independent of x and Q^2:

$$R\left[\frac{F^D(\xi, x, Q^2)}{F^{ND}(x, Q^2)}\right]_{\text{HERA}} = \frac{A_{\text{norm}} \cdot c_q}{\xi^{1+\epsilon}} = \frac{\text{constant}}{\xi^{1+\epsilon}} \tag{10}$$

3.3. *Predictions for the diffractive structure function at the Tevatron*

At the Tevatron, where high soft parton densities lead to saturation, A_{norm} must be renormalized by being replaced by

$$\text{Tevatron:} \quad A_{\text{renorm}} = 1/\int_{\xi_{min}}^{\xi=0.1} \frac{d\xi}{\xi^{1+\epsilon+\lambda}} \propto \left(\frac{1}{\beta s}\right)^{\epsilon+\lambda} \tag{11}$$

where we have used $\xi_{min} = x_{min}/\beta$ and $x_{min} \propto 1/s$. Thus, the diffractive structure function acquires a term $\sim (1/\beta)^{\epsilon+\lambda}$, and the diffractive to inclusive structure function ratio a term $\sim (1/x)^{\epsilon+\lambda}$. This prediction is confirmed by the CDF data on dijet production, where the x-dependence of the diffractive to inclusive ratio was measured to be $\sim 1/x^{0.45}$. [d]

4. Summary and conclusions

We have reviewed experimental data on soft and hard diffraction, concetrating on aspects that point to the partonic nature of the exchange (Pomeron) responsible for rapidity gap formation.

In soft diffraction, the exponential dependence of total [elastic] cross sections on $\Delta\eta'$ [$\Delta\eta$], the rapidity region in which there is [there is not] particle production, allows differential diffractive cross sections to be factorized into two terms: one representing the total cross section at the reduced energy squared, defined by $\ln(s'/s_0) = \sum_i \Delta\eta'_i$, and the other depending on $\sum_i \Delta\eta_i$ and interpreted as the gap formation probablity. By (re)normalizing the latter to unity and multiplying by a color factor κ (κ^n for n gaps) derived from gluon and quark color factors weighted by the corresponding inclusive pdf's at $Q^2 \approx 1$ GeV^2, cross sections for single, central, and multigap diffraction are obtained, which are in excellent agreement with experimental results. Prominent aspects of the data explained in this model are the

[d] In calculating the value r of the $\sim 1/x^r$ dependence, care should be taken to separately renormalize the sea gluon and sea quark contributions, and also consider the contribution of the valence quarks (Reggeon contributions).

M^2-scaling behavior and the independece on the number of gaps of the suppression of diffractice cross sections at high energies relative to Regge theory predictions.

In hard diffraction, the diffractive structure function is obtained by convoluting the inclusive $F(Q^2, x)$ with the parton density at $x = \xi$ at scale $Q^2 \approx 1$ GeV^2, identifying the gap formation probability term, and (re)normalizing the latter to unity at the Tevatron. Aspects of the data explained by this approach are the rise of the Pomeron intercept with Q^2 and the constant DDIS/DIS ratio versus x and Q^2 at HERA, as well as the falling as $\sim x^{-r}$ of the SD/ND ratio at the Tevatron. Also explained in this model is the factorization breakdown (and restoration for multigap diffraction!) between Tevatron diffracive structure functions and expectations from DDIS at HERA.

In conclusion, scaling and factorization aspects of the data have been phenomenologically interpreted in a parton model approach, in which diffraction is treated as a low x partonic interaction subject to color constraints.

References

1. J.D. Bjorken, *Phys. Rev.* **D47**, 101 (1993).
2. P.D.B. Collins, in *An Introduction to Regge Theory and High Energy Physics* (Cambridge University Press 1977).
3. K. Goulianos, in *Hadronic Diffraction: Where do we Stand?*, Les Rencontres de Physique de la Vallé d'Aoste, La Thuile, Aosta Valley, Italy, February 29 - March 6, 2004; e-Print Archive: hep-ph/0407035.
4. F. Abe et al. (CDF Collaboration), *Phys. Rev.* **D50**, 5535 (1994).
5. T. Affolder et al. (CDF Collaboration), *Phys. Rev Lett.* **87**, 141802 (2001).
6. D. Acosta *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **93**, 141601 (2004).
7. D. Acosta et al. (CDF Collaboration), *Phys. Rev Lett.* **91**, 011802 (2003).
8. K. Goulianos, *Phys. Reports* **101**, 171 (1983).
9. K. Goulianos and J. Montanha, *Phys. Rev.* **D59**, 114017 (1999).
10. K. Goulianos, *Phys. Lett.* **B358**, 379 (1995); Erratum-*ib.* **B363**, 268 (1995).
11. K. Goulianos, in *Diffraction in QCD*, Corfu Summer Institute on Elementary Particle Physics, Corfu, Greece, 31 Aug - 20 Sep 2001; e-print Archive: hep-ph/0203141.
12. E. Levin, in *An Introduction to Pomerons*, Preprint DESY 98-120.
13. R.J.M. Covolan, J. Montanha, and K. Goulianos, *Phys. Lett.* **B389** (1996) 176.
14. A. Petrukhin (H1 Collaboration), in *Measurement of the Inclusive DIS Cross Section at Low Q^2 and High x Using Events with Initial State Radiation*, presented at DIS2004, 14-18 April 2004, Slovakia.
15. F.P. Schilling (H1 Collaboration), in *Measurement and NLO DGLAP QCD Interpretation of Diffractive Deep-Inelastic Scattering at HERA*, submitted to 31st International Conference on High Energy Physics, ICHEP02, Amsterdam, The Netherlands, Jul. 24–31, 2001 (abstract 980).
16. J. Collins, *J. Phys.* **G28**, 1069 (2002); e-Print Archive: hep-ph/0107252.